Optimal Fisheries Investment under Uncertainty

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A full analysis of optimal fisheries investment strategies must take into account high levels of uncertainty in future fishery returns, as well as irreversibility of investment in specialized, nonmalleable fishing fleets. A stochastic optimization model is analyzed using dynamic programming to determine optimal policy functions for both fleet investment and fish stock management within an uncertain environment. The resulting policies are qualitatively similar to those found in the corresponding deterministic case, but quantitative differences can be substantial. Simulation results show that optimal fleet capacity should be expected to fluctuate over a fairly wide range, induced by stochastic variations in the biomass. However, the performance of a linear-cost risk-neutral fishery is fairly insensitive to variations in investment and escapement policies around their optimum levels, so that economic optimization is "forgiving" within this context. A framework of balancing upside and downside investment risks is used here to explain the roles of several fishery parameters in relation to optimal investment under uncertainty. In particular, the intrinsic growth rate of the resource and the ratio of unit capital costs to unit operating costs are found to be key parameters in determining whether investment should be higher or lower under uncertainty.

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Une analyse complète des stratégies d'investissement optimales dans les pêches doit tenir compte de la grande incertitude qui caractérise les revenus, de même que l'irréversibilité de l'investissement dans des flottilles de pêche spécialisées, non malléables. Nous analysons dans l'article qui suit un modèle d'optimisation stochastique à l'aide d'une programmation dynamique, afin de déterminer des fonctions de politiques optimales, tant pour l'investissement dans les flottilles que pour la gestion des stocks de poissons dans un climat d'incertitude. Les politiques qui en résultent sont qualitativement identiques à celles qui avaient été trouvées dans le cas déterministe correspondant, mais il peut exister d'importantes différences quantitatives. D'après les résultats de la simulation, on devrait s'attendre que la capacité optimale de la flottille varie dans une gamme assez étendue, par suite de variations stochastiques de la biomasse. Cependant, le rendement d'une pêche de coût linéaire et de risque neutre est assez insensible aux variations de l'investissement et aux politiques d'échappement autour de leurs niveaux optima, de sorte que l'optimisation économique est « indulgente » dans ce contexte. Afin d'expliquer les rôles de plusieurs paramètres de pêche par rapport à un investissement optimal dans des conditions d'incertitude, nous équilibrons les risques d'investissement en hausse et en baisse. Nous constatons, en particulier, que le taux de croissance intrinsèque de la ressource et le rapport entre coût en capital unitaire et coût d'opération unitaire sont les paramètres-clés quand il s'agit de déterminer si, dans des conditions d'incertitude, l'investissement devrait être plus élevé ou plus faible.

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THIS paper expands upon the fisheries investment model presented in Charles (1983a). That paper studied the comparative dynamics of optimal fisheries development in a deterministic world, where capital investment in the fishing fleet and harvest management ("investment in the resource") must be considered simultaneously. As in the related work of Clark et al. (1979), nonmalleable capital and the resulting irreversibility of investment played a key role.

In this paper, the irreversible investment problem is made even more realistic by recognizing that fisheries investment decisions must typically be made within an uncertain environment (c.f. Arrow and Lind 1970; Brock and Mirman 1978). In particular, uncertainty is incorporated here in the form of year-to-year stochastic resource fluctuations, so that future fish stock levels cannot be predicted in advance (although average values are known) — this is certainly a common

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feature of most fisheries. Primary emphasis is placed on examining (i) the appearance of an optimally managed stochastic fishery, (ii) the role of economic and ecological parameters in determining how uncertainty affects optimal investment, and (iii) the relative performance of deterministic and stochastic investment policies.

Questions of optimal fisheries investment under uncertainty have also been considered by Dudley and Waugh (1980) and McKelvey (1983). These interesting papers examine, respectively, the effects of several stochastic components in the optimal capacity problem and the optimal mix of "specialist" and "generalist" and "generalist" vessels in a fishing fleet. However, both studies make the simplifying assumption that the fish population in any year is independent of past harvests; this avoids the complexities involved in the joint investment problem but limits the number of fisheries to which their results can be applied. For recent reviews of other literature on fisheries management under uncertainty, see Andersen and Sutinen (1983) and Spulber (1983).

The Model

The stochastic model utilized here is a direct analog of the deterministic case discussed in Charles (1983a), to which the reader is referred for details. To summarize, the dynamics of the fish stock (R) and the fishing fleet (K) are given by

$$R_{n+1} = F(S_n)\cdot Z_n$$

$$K_{n+1} = (1 - \gamma)K_n + I_{n+1}$$

where in any year, the decision variables, escapement (S) and investment (I), are constrained by the current fish stock and capital stock, according to

$$R_n e^{-qTK_n} \le S_n \le R_n$$

$$I_{n+1} \ge 0.$$

The former reflects a constraint on intraseasonal harvesting effort, $0 \le E(t) \le K$, while the second constraint represents the irreversibility of investment. The lognormal random variables $\{Z_n\}$ are assumed to be independent and identically distributed, with mean value 1 ($E(Z_n) = 1$) and with log (Z_n) having variance σ^2. Hence, recruitment R follows a lognormal probability distribution with mean $F(S)$, where S is the previous year's escapement and $F(\cdot)$ is the corresponding deterministic stock-recruitment function.

Use of the lognormal distribution is motivated by two factors: (1) it is the natural distribution to reflect the large number of independent multiplicative effects facing the growth of fish from the egg to the adult stage and (2) it reproduces qualitative features of fisheries data, where one often sees a large number of low to medium recruitments and occasional very large recruitments.

As in most other fisheries management models, it is assumed that yearly recruitment and escapement are observable, although in practice, errors in measuring the biomass certainly add to the overall level of uncertainty and complicate the optimization analysis.

The resource management problem can be summarized as follows.

1) Given initial biomass R and capacity K, an optimal end-of-season escapement S^* must be determined.

2) The desired capacity for the following year is determined, and payment is made for the corresponding investment I^*. (A 1-yr delay in bringing new investment online is assumed, contributing to the resource manager's uncertainty in a stochastic environment.)

3) Harvesting and stochastic population dynamics occur, so that by the start of the next season, the biomass is $R' = F(S^*)\cdot Z$ for some value of the random variable Z.

4) Depreciation and investment take place, producing a capacity $K' = (1 - \gamma)K + I^*$ next season.

It is assumed throughout this paper that the social resource manager is risk neutral, that the fishery faces perfectly elastic demand (with given constant selling price p), and that costs are linear, with unit cost of harvesting effort c and unit cost of capital δ. (These assumptions will be relaxed in future work. Andersen (1982) and Pindyck (1982) considered questions of optimal harvesting under price variability but did not address the capital investment problem.)

The yearly rents accruing to the fleet, as a function of recruitment, capacity, escapement, and investment, are as in Charles 1983a:

$$\pi(R,K,S,I) = p(R - S) - (c/q)\log(R/S) - \delta I.$$

The fishery optimization problem is as before, with the exception that now, future fish stock sizes are averaged over a probability distribution. Specifically, next season's recruitment follows the lognormal density $R_{n+1} \sim \phi_{F(S),\sigma}(\cdot)$ with

$$\phi_{\bar{R},\sigma}(R) = [\sqrt{(2\pi)}\sigma R]^{-1} \times \exp\{-(\log R - \log \bar{R} + \sigma^2/2)^2/2\sigma^2\}$$

where $\bar{R}$ is the mean, given by $\bar{R} = F(S)$, and the variance is $\bar{R}^2(e^{\sigma^2} - 1)$.

The dynamic programming equation corresponding to this problem is then given by

$$(1)\quad V(R,K) = \max_{R\cdot\exp(-qTK)\le S\le R}\ \max_{I\ge 0} \{\pi(R,K,S,I) + \alpha E[V(R', (1 - \gamma)K + I)]\}$$

where (R,K) is the "state" this year, (S,I) are the controls (decision variables), R' is next year's recruitment (lognormally distributed as above), and α is the discount factor.

Heuristic Analysis and Numerical Method

Heuristic Analysis

Equation 1 is identical to the corresponding equation in Charles 1983a, except that the future value of the fishery is now averaged over possible future recruitments. The heuristic analysis proceeds in a similar manner, producing the following optimality results for the target investment curve $K = h(S)$, the target escapement curve $S = s(K)$, and the actual (feasible) investment and escapement, $I^*(S,K)$ and $S^*(R,K)$, respectively:

$$(2)\quad E\{V_K[R, (1 - \gamma)K + I]\} = \delta/\alpha \quad \text{or } I = 0 \text{ if } E\{V_K[R, (1 - \gamma)K]\} < \delta/\alpha$$

$$(3)\quad I^*(S,K) = \max[h(S) - (1 - \gamma)K, 0]$$

$$(4)\quad \alpha[F'(S)/F(S)]\cdot E\{R'\cdot V_R[R', (1-\gamma)K + I^*(S,K)]\} = p\left(1 - \frac{x_0}{S}\right)$$

$$(5)\quad S^*(R,K) = \begin{cases} R\cdot\exp(-qTK); & R > s(K)\exp(qTK) \\ s(K); & R \text{ intermediate} \\ R; & R < s(K) \end{cases}$$

where the expectation over R is with respect to the lognormal density $\phi_{F(S),\sigma}(\cdot)$, $x_0 = c/pq$ represents bionomic equilibrium, $K = h(S)$ solves $\underset{Z}{E}\{V_K(F(S)\cdot Z,K)\} = \delta/\alpha$, and $S = s(K)$ solves equation 4.

In Charles (1982) it is shown that the resulting optimal policy curves are qualitatively similar to their deterministic analogues. The important questions, then, concern the behavior of optimally managed stochastic fisheries and the extent to which randomness affects the quantitative aspects of the optimal policies.

Numerical Method

Based on the above heuristic results, numerical methods have been developed to solve equation 1, using policy iteration to determine the optimal $S = s(K)$ and $K = h(S)$ curves. The approach used here extends methods of Ludwig (1979) and Ludwig and Walters (1982); it is more refined than that used in Charles (1983a), involving the evaluation of integrals over an infinite interval to determine the expected value function $E[V(\cdot,\cdot)]$.

The policy iteration algorithm proceeds as follows. First, an initial guess is made for the policy functions $s(K)$ and $h(S)$. For these functions, the value function $V(R,K)$ and its first partial derivatives V_R and V_K are determined simultaneously over all points on a discrete grid in the biomass/capacity plane. (An 8 by 8 grid was used in all cases, except for those involving the whale fishery (see Numerical Results below) where a 12 by 12 mesh was necessary to provide suitable accuracy.) The values of V, V_R, and V_K implicitly define a differentiable surface in $R-K$ space. It is necessary to extrapolate this surface beyond the limits of the discrete grid, so as to include the entire range of possible biomass values, $(0,\infty)$; the extrapolation procedures are described in Charles (1982). (In fact, for reasons of numerical accuracy and convenience, the method uses $x = \log(R)$ in place of R as a state variable. Naturally, this change of variables does not affect the final results.)

The next step is to improve upon the initial policy functions by inserting the newly found values of V, V_R, and V_K into the optimality equations 2 and 4. Solving these equations numerically produces new policies $s(K)$ and $h(S)$, which can be expected to outperform the policies used at the previous step. Repeating the process using these new policies results in an iterative approach to the overall optimum. This scheme performed well for all cases discussed below, although convergence was rather slow in the absence of depreciation ($\gamma = 0$).

Numerical Results

The numerical methods described above permit a full solution to the management problem of determining optimal investment and escapement policies within an uncertain environment. As in Charles (1983a), the Australian Gulf of Carpenteria banana prawn fishery (Clark and Kirkwood 1979) and the aggregated pelagic whaling fishery (Clark and Lamberson 1982) are examined. In most cases, the general form of the prawn fishery has been used as the primary source of data, but the parameters have been varied to study comparative dynamics. The base parameters for both fisheries are as in Charles (1983a). (Although the model used here is simple in comparison with most real-world fisheries, there is nothing specific in its structure to detract from fairly wide applicability. Hence, with suitable caveats, the general results should hold also in other fisheries and indeed other renewable resource industries.)

The underlying stock-recruitment function $F(S)$, representing the average recruitment for a given escapement S, is taken to be either $F(S) = aS/(1 + aS/b)$ or $F(S) = aS\cdot e^{-aS/eb}$ for the Beverton–Holt and Ricker cases, respectively. The parameters a and b then represent the maximum productivity and the maximum possible mean recruitment. For the prawn fishery, the fairly high, but realistic, value of $\sigma = 0.58$ was used in most cases for the uncertainty parameter (representing the standard deviation of the logarithm of recruitment). This value of σ is the maximum likelihood estimate obtained by fitting a lognormal distribution to prawn recruitment data from G. P. Kirkwood (C.S.I.R.O. Division of Fisheries, Cronulla, Australia, personal communication), with the mean value of the distribution simply equated to the sample mean of the data.

In general, the approach used here is to compare optimal policy functions for a fishery subject to a fairly high degree of uncertainty ($\sigma = 0.58$) with the corresponding optimal policies in the absence of uncertainty ($\sigma = 0$). The latter deterministic results correspond to those of Charles (1983a) but are obtained using the more accurate numerical method described above.

The first subsection below describes the appearance and behavior of an optimally managed stochastic fishery. The following subsections examine the effect of several key fishery parameters in determining the role of uncertainty in optimal fisheries management. In particular, it is of interest to study whether investment increases or decreases with increasing uncertainty, and how the outcome is affected by (i) the intrinsic biomass growth rate, (ii) the capital cost (relative to variable costs), (iii) the discount rate, and (iv) the depreciation rate.

Appearance and Behavior of a Stochastic Fishery

Figure 1a depicts the optimal investment and escapement policy functions $h(S)$ and $s(K)$ for the base case prawn fishery with $\sigma = 0.58$. As discussed in the previous section, the curve $h(S)$ represents the target fleet capacity for next season, given escapement S this year. In other words, it is desirable to purchase $I^*(S,K) = \text{Max}\,[h(S) - (1-\gamma)K, 0]$ in new fleet capacity, even though stochastic recruitment next season can only be predicted roughly (i.e. in the mean) at the time of ordering the investment. The $s(K)$ optimal escapement curve is entirely analogous to its deterministic counterpart.

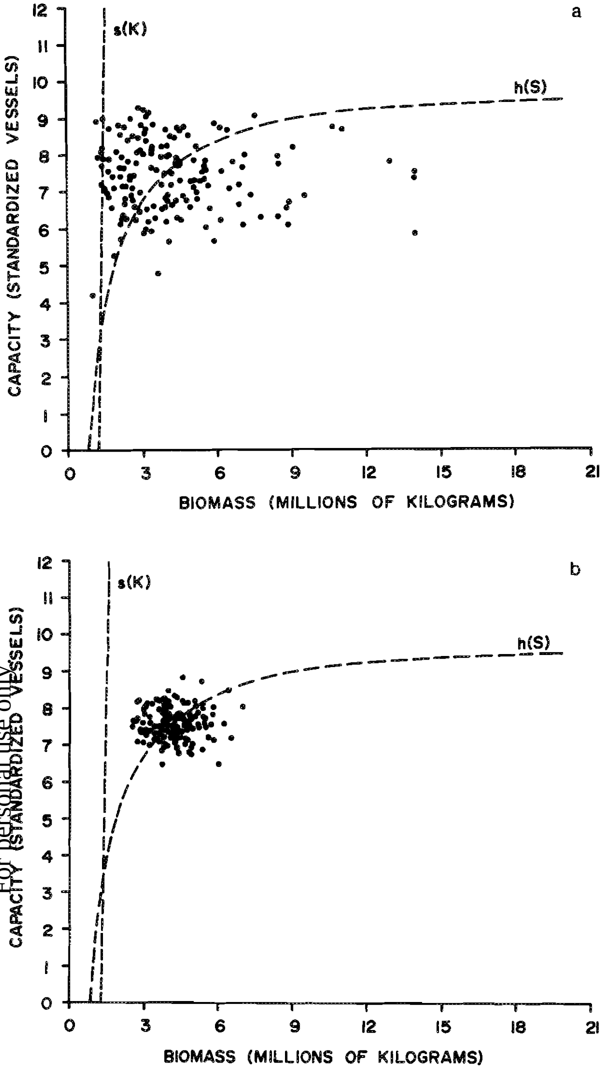

FIG. 1. Optimal policy functions, $h(S)$ and $s(K)$, for the stochastic base case prawn fishery, with uncertainty parameter (a) $\sigma = 0.58$ and (b) $\sigma = 0.2$. In addition, the steady-state distribution for this optimally managed fishery is approximated by the endpoints of 160 40-yr simulations, beginning each time at the quasi-equilibrium point (4.3×10^6, 7.75) (see text for details).

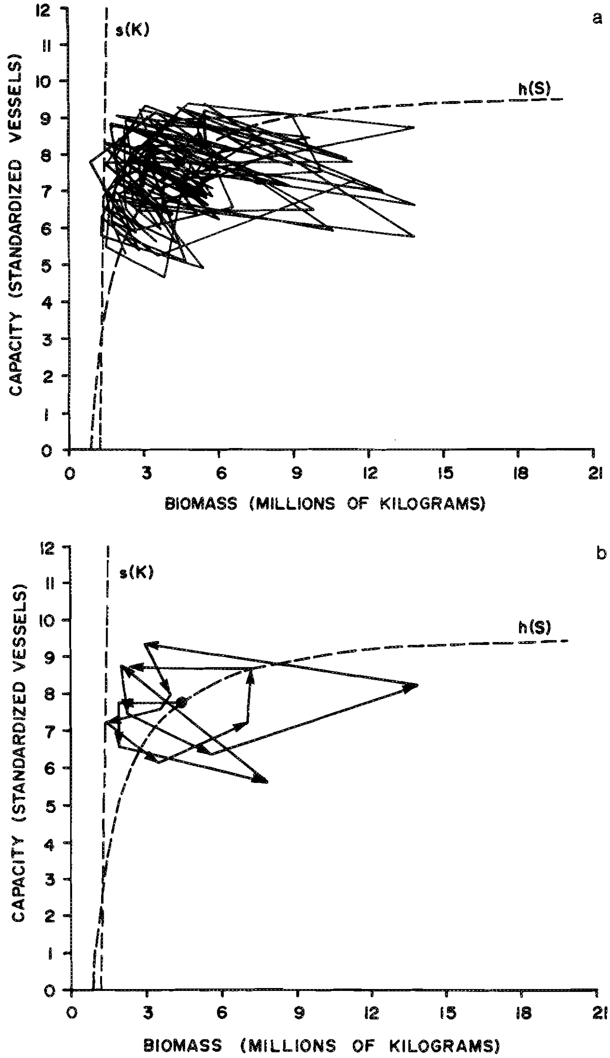

FIG. 2. Optimal policy functions for the base case stochastic prawn fishery, together with sample paths showing the effect of stochastic fluctuations on optimal management of the fishery. (a) Eight 20-yr sample paths, each beginning from the quasi-equilibrium point. (b) A single 15-yr outcome. The arrows join (S,K) points; the processes of depreciation, investment, recruitment, and harvesting occur successively between each pair of points.

The point $P(4.3 \times 10^6, 7.75)$ marked in Fig. 1a would be the long-run equilibrium point for the fishery if there were no random fluctuations. It is referred to here as a "quasi-equilibrium point"; deterministic fish and fleet dynamics tend to push the fishery towards this point, but stochastic perturbations prevent actual convergence. In fact, as pointed out by May et al. (1978) and Spulber (1983), deterministic equilibrium points translate into steady-state probability distributions in the stochastic case. In the present two-dimensional model, any steady state would also be two-dimensional, although the existence of such an equilibrium distribution has not been explicitly examined here. Instead, a steady-state distribution has been approximated by plotting the end points of a large number (160) of 40-yr fishery simulations, each emanating from the quasi-equilibrium point P; these end points are depicted in the figure.

The cloud of points shown in Fig. 1a can be interpreted as follows: the denser the points in a given region of the $S-K$ plane, the more likely is the fishery to lie in that region (i.e. to have that escapement and that capacity) over the long term. One can observe a considerable spread both in biomass and fleet capacity values about the quasi-equilibrium point. The spread in biomass values is due simply to the stochastic nature of the resource. Variation in the capital stock, on the other hand, is an induced phenomenon; fluctuations in recruitment lead directly to variations in escapement, which in turn cause dispersion in fleet capacity, through the investment function $K^* = h(S)$. This effect will be even more pronounced with slower growing stocks, where particularly good or bad escapement levels will tend to influence the fishery for longer periods of time and will therefore have a greater effect on desired fleet capacity.

Since the resource is fairly fast-growing ($a = 42$), few points are found at low ($S < s(K)$) escapement levels. In fact, the distribution of points resembles a lognormal distribution in the S direction, truncated below at $S = s(K)$. This is unlikely to be the case precisely, however, since (S,K) rather than (R,K) points have been plotted, resulting in a tighter distribution. (Relatively high (lognormally distributed) R values are reduced by fishing pressure to comparatively low S values.) In addition, the spread of points in the S direction can be seen to be smaller at high capacities, since in this case fishing effort is sufficient to reduce even high recruitments down to escapements fairly near the $s(K)$ curve.

Figure 1b shows the effect of reducing the noise level from $\sigma = 0.58$ to $\sigma = 0.2$ in the prawn fishery. As expected, the steady-state distribution collapses to within a much smaller neighborhood of the quasi-equilibrium point. One would expect stochastic effects to be relatively unimportant at such low σ values; however, as shall be seen, the values of σ that can be considered "low" depend on the other fishery parameters. In the whale fishery, $\sigma = 0.2$ can be a substantial level of noise.

To illustrate more vividly the actual process of managing a fishery in a stochastic environment, Fig. 2a depicts a set of eight 20-yr sample paths for the optimally managed base case prawn fishery, with the recruitment chosen each year from a lognormal ($\sigma = 0.58$) density centred on $F(S)$, where S is the previous year's escapement. Lines are drawn joining successive (S,K) points, beginning at the quasi-equilibrium point. As above, it can be seen that optimal risk-neutral management results in considerable variation in the fleet capacity, as well as the biomass, over time.

Figure 2b shows this effect in more detail for a single 15-yr realization of the fishery's development. The four processes of (stochastic) recruitment, harvesting, investment, and depreciation combine to determine the movement from one (S,K) point to the next, governed by the policy curves. Since the resource is fast-growing and highly variable in this example, there is no apparent trend to return to the quasi-equilibrium point.

Cost of Capital

Consider Fig. 3 in which the optimal capacity curves $h(S)$ are shown for fisheries with $a = 14$ throughout, but with varying levels of uncertainty and unit capital cost.

To concentrate on the investment problem, the optimal escapement curves $s(K)$ have been omitted from Fig. 3. Typically, the optimal escapement level was found to be fairly insensitive to the degree of uncertainty for most of the parameter combinations considered, a result in accordance with those of other researchers. There are, however, cases where uncertainty does affect the $s(K)$ curves; these are discussed

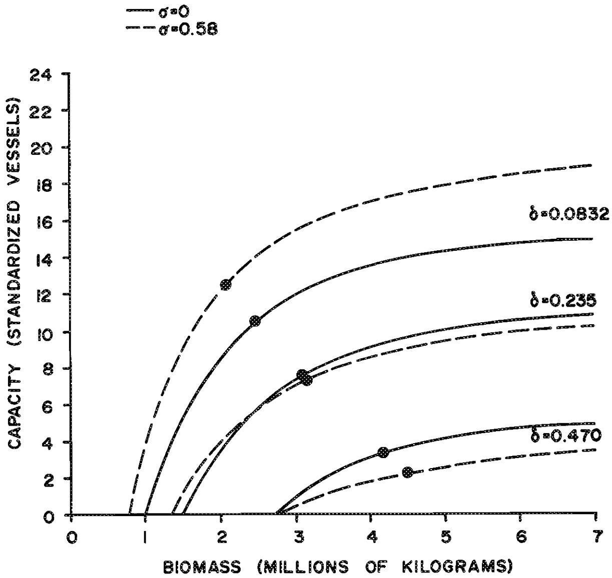

FIG. 3. Deterministic ($\sigma = 0$) and stochastic ($\sigma = 0.58$) optimal fleet capacity functions, $h(S)$, for an $a = 14$ prawn fishery with three values of the unit capital cost: $\delta = 0.0832$, 0.235, and 0.470 (Australian dollars $\times 10^6$).

below. For the remainder of this paper, $s(K)$ curves are shown only in such cases.

It was seen in the deterministic analysis of Charles (1983a) that the important cost parameter in the investment problem is neither capital cost nor operating cost alone, but rather the ratio of the two. Specifically, a useful quantity to study appears to be δ/cT, the ratio of unit capital cost to maximum yearly operating cost (per unit of capital). In a sense, this measures the capital intensity of the fleet, since the present value of maximum total costs per unit of capacity is

$$\delta + \beta cT = cT[(\delta/cT) + \beta]$$

where

$$\beta = \alpha \sum_{n=0}^{\infty} [\alpha(1-\gamma)]^n = \frac{\alpha}{1 - \alpha(1-\gamma)}$$

allows for discounting and depreciation. The important aspect of Fig. 3 is the relative position of the $h(S)$ curve between the deterministic and stochastic cases, as this ratio of costs, δ/cT, varies. However, since cT is fixed here, it suffices to speak in terms of changes in the capital cost. It can be seen that at low capital costs, the optimal capacity is substantially higher in a fluctuating environment, while at high unit capital costs the reverse is true. At the intermediate level $\delta = \$0.235$ million, the optimal $h(S)$ curves in the deterministic and stochastic cases exhibit a crossover, so that the introduction of randomness increases optimal investment at low biomass levels while decreasing investment at higher stock sizes.

With regard to the $s(K)$ policy functions, the slower growing $a = 14$ fishery shows a slightly greater effect of randomness on the optimal escapement levels than in the $a = 42$ base case: this is indicated for the $a = 14$, $\delta =$

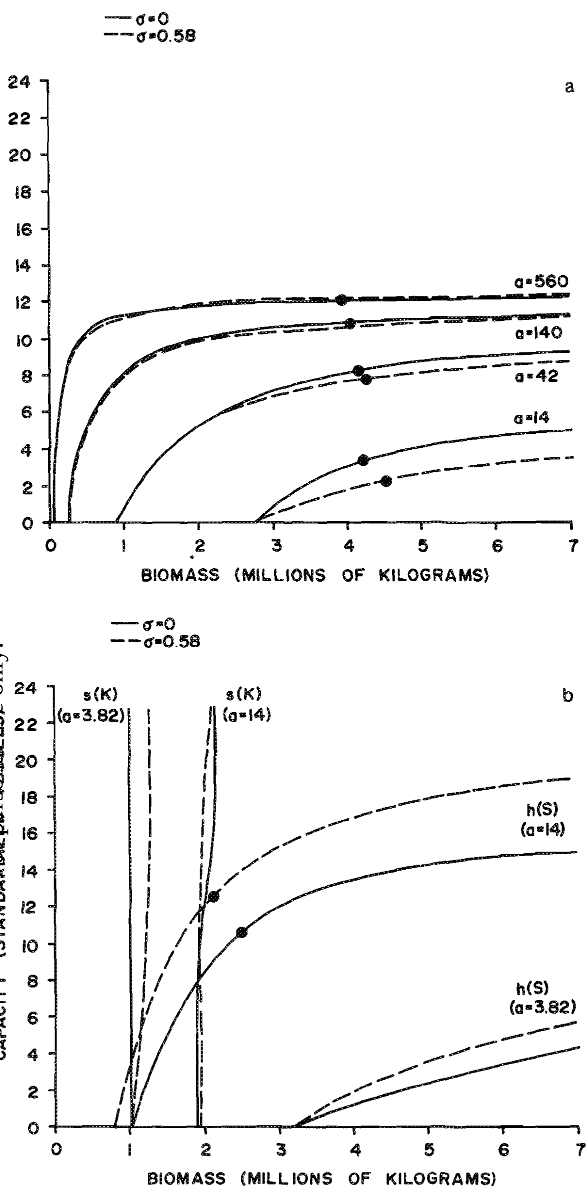

FIG. 4. Joint effect of the biomass growth rate and the level of uncertainty. (a) Optimal capacity functions for the base case prawn fishery with growth rates a = 14, 42, 140, and 560. (b) A lower level of unit capital cost is assumed (δ = A\$0.0832 million rather than δ = A\$0.47 million), and the values a = 3.82 and a = 14 have been used as possible fish productivity levels. In each case, deterministic (σ = 0) and stochastic (σ = 0.58) cases are considered.

$0.0832 $\times 10^6$ fishery in Fig. 4b and is discussed further below.

The optimal capacity results described here can be explained by considering two opposing effects: the "downside" risk of suffering idle excess capacity in bad years and the "upside" risk of lacking sufficient capacity to take full advantage of good years. Ceteris paribus, the optimal fleet capacity increases with uncertainty given a relatively low ratio of capital to operating costs, since the balance of these risks is tilted towards the upside benefits of investing in extra capacity (at relatively low cost) to take advantage of exceptionally high recruitments. However, if unit capital costs are sufficiently high relative to operating costs, investment will decrease with the level of uncertainty. In this case the downside risk of more frequent bad years (when there is little or no return on the expensive investment) outweighs the advantages of having extra capital available to profit from good years. In the intermediate case, it appears that the role of uncertainty depends on the escapement level; at high biomass levels, the variance in recruitment is also high, so that the downside problem predominates. Hence, investment is lower under uncertainty; the balancing act tilts towards caution in investment. On the other hand, if escapement is already relatively low, and the stock tends (in the mean) to grow reasonably rapidly, then the potential benefits to extra (upside) investment outweigh the downside risk. Increased investment under uncertainty becomes optimal. Where the crossover will occur, if it does at all, seems rather difficult to predict. Indeed in most cases where a crossover appears, the difference between the $h(S)$ curves in the deterministic and stochastic cases tends to be small.

Productivity of the Resource Stock

Figure 4a shows deterministic and stochastic optimal capacity functions for each of four possible values of the growth rate parameter, with other parameters as in the base case fishery. (The value a = 560, corresponding to very high fish stock productivity, was chosen to approximate a situation of independence between recruitment and escapement.)

Examining Fig. 4a, one can observe a uniform progression from high to low growth rates. When $a = \infty$, it is shown in Charles (1982) that the optimal capacity must increase with the level of uncertainty. The case a = 560 follows this result, at least for reasonably high stock levels. However, with a = 140, the optimal capacity is lower under uncertainty, and this effect increases as the growth rate is decreased to a = 42 and then to a = 14. (As before, in each case, there is little difference between the stochastic and deterministic optimal escapements.)

If the unit capital cost is reduced, in this case to \$0.0832 million, results remain qualitatively unchanged. Figure 4b shows the deterministic (σ = 0) and stochastic (σ = 0.58) optimal policy functions for the two cases a = 14 and a = 3.82, with the lower capital cost. When natural mortality is taken into account the latter a value corresponds to a maximum net growth rate of 4% annually. This value was chosen to equal that of the whale fishery for comparative purposes discussed below. In this case, even at such low productivity, the unit capital cost is sufficiently low that the optimal capacity remains positive for both the deterministic and stochastic cases. The target capacity is generally higher under uncertainty, although there is a very slight crossover in the $h(S)$ curves, a result that is returned to in discussion of the whale fishery. As the biomass growth rate is increased to a = 14, the optimal capacity under uncertainty rises even further above its deterministic counterpart.

While optimal policies can be determined for the a = 3.82

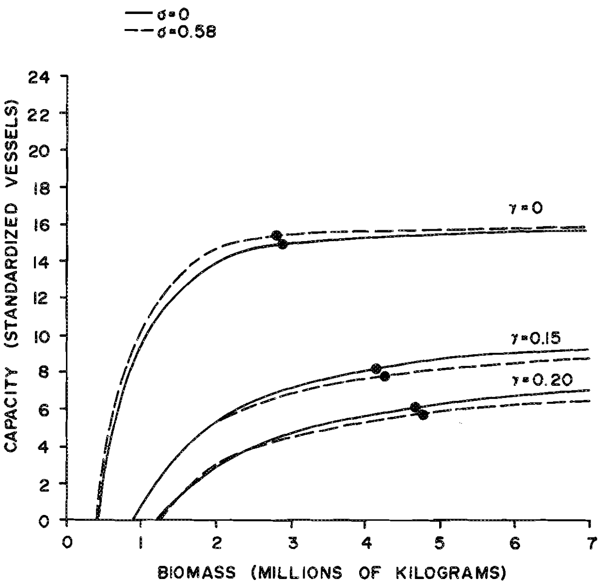

FIG. 5. Depreciation rate and uncertainty: the optimal deterministic and stochastic capacity functions are shown for each of γ = 0, 0.15, and 0.20 for the base case prawn fishery.

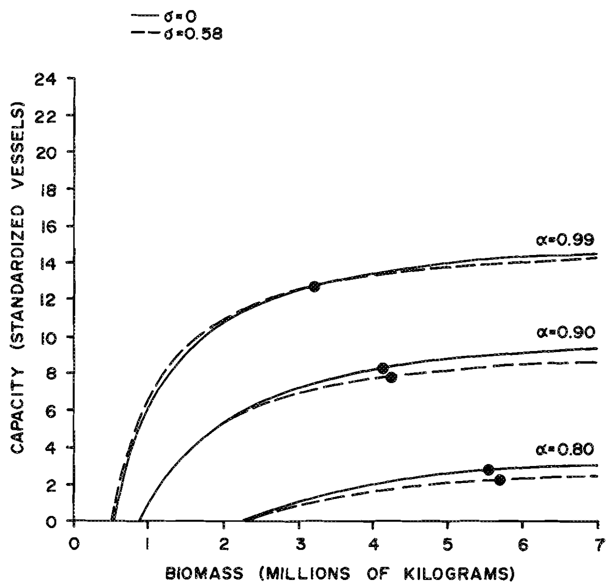

FIG. 6. Discount rate and uncertainty: the optimal deterministic and stochastic capacity functions are shown for each of α = 0.80, 0.90, and 0.99 (with respective discount rates r = 25, 11, and 1%) in the case of the prawn fishery.

fishery, in fact this fishery would not be economically sustainable in a deterministic environment; the equilibrium occurs at $R = S = 0.28 \times 10^6$, which is well below the rent-dissipating bionomic equilibrium. Hence, no fishery will exist unless the resource is artificially enhanced or economic conditions improve significantly. In the stochastic case, however, there is always a possibility of the biomass reaching a sufficiently high level to warrant investment in the fishery. In the example shown here, such an occurrence will be rare, but amongst the world's fisheries, there will likely be many that should follow such a pattern of periodic development, responding to occasional exceptionally high recruitments.

With regard to optimal escapement levels, when $a = 3.82$, $s(K) = x_0$ (for all K) in the deterministic case, while $s(K) > x_0$ for $\sigma = 0.58$. Hence, the fishery should be driven towards its zero-profit level in the deterministic case but should be somewhat more conservationist under uncertainty. The $a = 14$ optimal escapement curves exhibit rather complicated behavior, with the stochastic $s(K)$ curve lying above the $\sigma = 0$ curve, except in an intermediate range of fleet sizes (between $K = 10$ and $K = 24$). This intermediate phase appears to be caused by the considerable variation in the fleet investment policies with uncertainty, since $I^*(S,K)$ enters into the optimality equation determining $s(K)$. However, in any case, the maximum difference between the $s(K)$ curves with $a = 14$ is only $\Delta s = 0.15 \times 10^6$.

The interplay between the degree of randomness and the intrinsic growth rate can be explained by appealing to the downside versus upside argument discussed above. The slower growing the resource stock, the greater the connection between recruitment and the previous season's escapement. In effect, the memory of the system is longer, so that both particularly low and particularly high recruitments will tend to be more persistent in the fishery (although, of course, stochastic fluctuations will cause deviations from this trend). This has implications for both the downside and upside aspects of the investment question. On the one hand, the downside risk for new investment will be greater, since the probability of suffering a series of bad years is increased. On the other hand, the upside benefits of extra fleet capacity are reduced because a high recruitment will tend to persist over several seasons and can therefore be harvested at a more leisurely pace, using less capital. The situation is reversed with fast-growing stocks, where the high recruitments produced in good years must be utilized immediately or forever lost, and hence, there is a strong incentive to invest in additional capital. On balance, therefore, the stochastic optimal capacity will always exceed its deterministic counterpart if recruitment is independent of past escapements ($a = \infty$), but this effect will decrease and likely reverse itself as the intrinsic growth rate decreases and downside risks begin to outweigh upside benefits. In fact, the effect of uncertainty on optimal investment depends jointly on the biomass growth rate and the capital cost to operating cost ratio (see Discussion for a consideration of this rather complex interaction).

Depreciation Rate

Figure 5 shows that in the $a = 42$ case, the depreciation rate plays a role similar to that of the unit capital cost described above. In the absence of depreciation ($\gamma = 0$), the optimal capacity under uncertainty exceeds that of the deterministic case. Since capital is infinitely long-lived in such a case, the effective yearly rental cost of capital is relatively low. Hence, the downside risk of an increased capital stock (above that of the deterministic case) is relatively small. As the rate of depreciation increases, the effective time horizon

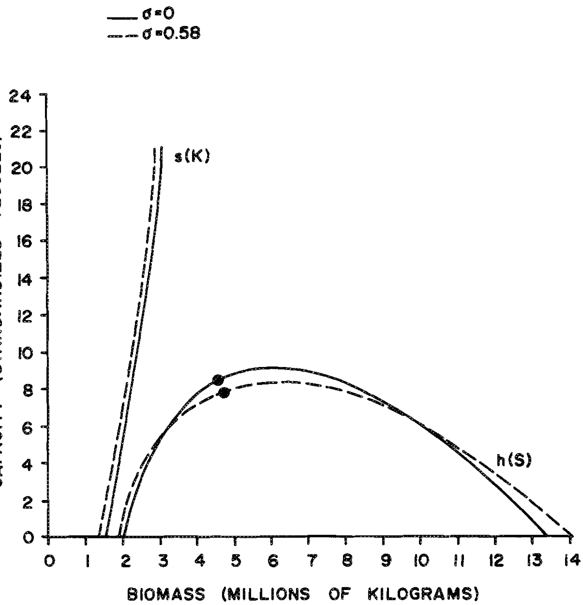

FIG. 7. Ricker stock-recruitment and uncertainty: the optimal policy functions $h(S)$ and $s(K)$ are shown for the prawn fishery with Ricker parameters $a = 3.172$ and $b = 7.0 \times 10^6$ and uncertainty $\sigma = 0$ and 0.58.

for a given unit capacity is shortened, so that the upside benefits of an extra unit of capacity are reduced, since there are likely to be fewer years in which the fishery could take advantage of a higher level of capacity. Thus, it is not surprising to see that the optimal capacity decreases with uncertainty when $\gamma = 0.15$ or $\gamma = 0.20$. In each of the three cases, the optimal escapement was insensitive to the level of uncertainty.

The effect of depreciation on this base case fishery can be summarized as follows: the lower the depreciation rate, the more likely is investment to be higher under uncertainty. In other words, the difference: "(stochastic optimal capacity) − (deterministic optimal capacity)" increases as the depreciation rate decreases.

With a slower growing ($a = 14$), lower capital cost ($\delta =$ \$0.0832 million, $\delta/cT = 2.0$) fishery, Charles (1983a) found that for the deterministic case, optimal investment levels can actually increase with the depreciation rate, if escapement is sufficiently high. It was argued that this result was due to an incentive to harvest the resource quickly, before the fleet depreciates. If capital is relatively cheap, this incentive outweighs the effective increase in unit capital cost due to depreciation. However, the same behavior does not carry over to the corresponding stochastic fishery; results obtained for this fishery indicate that although investment levels are uniformly higher with $\sigma = 0.58$ than in the deterministic case, optimal capacity decreases as the depreciation rate is increased, for all escapement values. This may be due to one of two reasons: either (i) the stochastic investment levels are already sufficiently high that the resource can be harvested as rapidly as necessary or (ii) stochastic fluctuations add sufficient unpredictability to the fishery that extra investment, to allow more rapid harvesting in the face of depreciation, is not warranted.

Discount Rate

Figure 6 depicts the optimal policy curves with and without randomness for three values of the discount factor (α) and corresponding discount rate ($r = [(1 - \alpha)/\alpha] \times 100\%$).

It can be noted that as the rate of discounting is increased (i) the relative difference between optimal deterministic and stochastic fleet capacities increases and (ii) the region of escapements for which optimal investment is greater under uncertainty diminishes. Hence, an increase in the discount rate is similar in effect to an increase in unit capital cost, increasing the downside risk of investment under uncertainty. However, it appears that at least for these parameter combinations, there is relatively little interplay between the discount rate and the level of uncertainty over a broad range of discount rates. Of course, the role of the discount rate in resource management is a complicated one, and these limited results should not be extrapolated too far.

Ricker Stock-Recruitment

If Beverton−Holt population dynamics are replaced by a Ricker reproduction function, the heuristic analysis and the results of Charles (1983a) suggest that the optimal investment curve $h(S)$ should mimic the Ricker form, rising to a peak and then declining to zero. Figure 7 confirms this qualitative behavior. In terms of the future of the fish stock, very large escapements are as bad as very small ones; if $K = 0$ but escapement $S > 14.0 \times 10^6$, the expected value of a unit of investment is less than its cost and, hence, $I^* = 0$.

Comparing the deterministic and stochastic optimal capacity functions, one can see that if the future of the fish stock is relatively bright ($3.25 \times 10^6 < S < 9.75 \times 10^6$), then investment is lower under uncertainty, while the reverse is true if expected future stock sizes are relatively small. This is equivalent to a single crossover in the $h(S)$ curves for the Beverton−Holt case (e.g. compare the $a = 14$, $\delta =$ \$0.235 $\times 10^6$ results shown in Fig. 3) and can be explained by examining the interaction of upside and downside investment risks, as above.

Whale Fishery

To this point, various modifications of the prawn fishery data have been considered. To check the robustness of the above results, this subsection examines the effect of uncertainty on the base case whaling fishery. Figure 8 shows the optimal capacity and escapement curves for the cases of $\sigma = 0$, $\sigma = 0.1$, and $\sigma = 0.2$, which cover a reasonable range of uncertainty for the aggregated Antarctic whale stocks.

It can be seen that increasing the level of uncertainty decreases the optimal capacity for relatively low stock sizes ($S < 360 \times 10^3$) but increases optimal capacity for high biomass levels. In other words, investment under uncertainty should respond to the state of the fishery; if the whale stock is particularly abundant, it is worthwhile taking advantage of

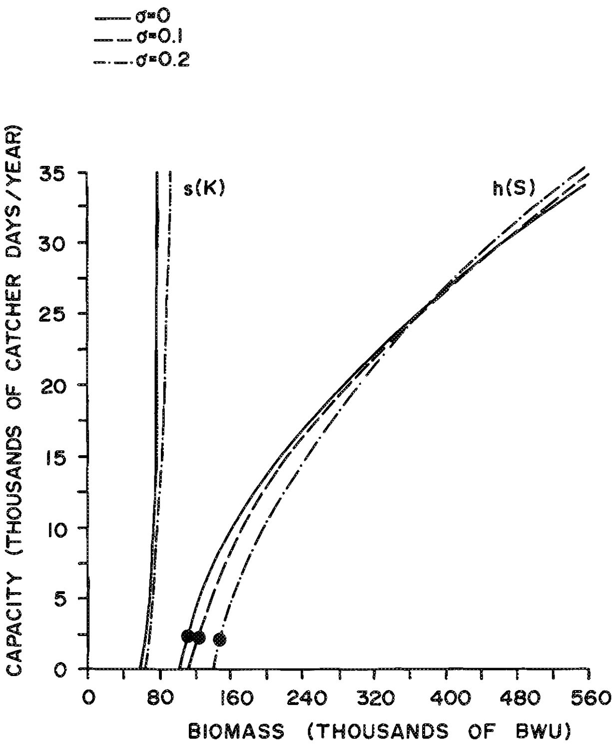

FIG. 8. Whale fishery under uncertainty: the optimal policy functions $h(S)$ and $s(K)$ are shown for each of the uncertainty levels $\sigma = 0$, 0.1, and 0.2 (the $s(K)$ curves for $\sigma = 0$ and $\sigma = 0.1$ differ negligibly).

good years by investing in capacity above the deterministic level, while for lower stock sizes a more conservative investment policy is to be preferred. However, the deterministic equilibrium point for the whale fishery lies at a low biomass level ($S = 114 \times 10^3$, $K = 2250$), so that even with $\sigma = 0.2$ one will rarely see escapements $S > 360 \times 10^3$ in the long term. On the other hand, the unexploited deterministic equilibrium lies at $S = 462 \times 10^3$. Hence, for this example the optimal stochastic policy faced with a virgin stock calls for an initial investment above the deterministic optimum but an investment level that is generally lower under uncertainty in the long term.

Note that both the maximum biomass growth rate and the ratio of unit capital cost to maximum yearly variable cost ($\delta/cT = 2.0$) for the base case whale fishery are the same as for a "prawn fishery" with $a = 3.82$ and $\delta = \$0.0832$ million (by design). Comparing Fig. 8 with results for that particular prawn fishery case, shown in Fig. 4b, one can see that the optimal policy curves are similarly behaved in most respects. In both cases, there is a crossover between the deterministic and stochastic $h(S)$ curves. However, the location and extent of the crossover is quite different in the two fisheries, possibly due either to the difference in noise levels being considered or to the difference in carrying capacity between the two fisheries. The latter, represented by the solution of $F(S) = S$, is relatively large for the whale fishery ($462 \times 10^3 \gg x_0 = 55 \times 10^3$) but small for the prawn fishery described above

TABLE 1. Expected value function $V(R,K)$ evaluated in a $\sigma = 0.58$ stochastic environment, using the appropriate optimal stochastic policies. Recruitment is given in millions of kilograms, capacity K in standardized vessels, and value in millions of Australian dollars.

	Recruitment							
Capacity	0.20	0.50	1.00	2.00	3.00	4.50	7.00	20.0
0.0	4.61	5.04	5.39	5.91	6.26	6.60	6.92	7.43
3.0	4.75	5.21	5.60	6.12	6.58	7.10	7.74	9.81
6.0	4.89	5.38	5.81	6.33	6.85	7.53	8.44	11.9
9.0	5.02	5.54	6.01	6.54	7.08	7.90	9.04	13.7
12.0	5.15	5.69	6.19	6.76	7.29	8.21	9.55	15.3
15.0	5.27	5.84	6.36	6.97	7.50	8.48	9.99	16.6
18.0	5.39	5.98	6.53	7.17	7.70	8.69	10.4	17.8
21.0	5.50	6.12	6.68	7.35	7.88	8.87	10.7	18.8

($0.28 \times 10^6 < x_0 = 1.0 \times 10^6$). In any case, the general result appears to be that investment is either zero or decreases with uncertainty for most reasonable escapement values in these fisheries; only in the case of exceptionally high escapements is it optimal for society to invest in fleet capacity above the deterministic optimum (assuming a low unit capital cost).

The primary difference between these results and others presented above is the fact that the crossover in $h(S)$ curves has been reversed, so that investment is now higher under uncertainty at high resource stock levels. The opposite effect was explained above by noting that biomass fluctuations, and the resulting downside risk, are relatively more important with large fish stocks. However, these results indicate that the explanation depends on resource productivity; when the fish population is very slow-growing, with a long inherent "memory," the downside risk diminishes as the stock grows, so that for an abundant stock, the optimal fleet capacity may actually increase with the level of fluctuations.

The optimal escapement for the whale fishery rises with increasing uncertainty, as in the $a = 3.82$, $\delta = \$0.0832 \times 10^6$ prawn fishery discussed above. It appears that management of a very slowly growing fishery should be more conservationist the higher the level of uncertainty; this is in accordance with previous research results. However, the difference between the $\sigma = 0$ and $\sigma = 0.2$ $s(K)$ curves is never very great, particularly for K values near the quasi-equilibrium point.

A particularly interesting result in comparing the deterministic and stochastic whale fisheries is the location of these quasi-equilibrium points: when $\sigma = 0.2$, the whale stock steady state is centered on $S = 148 \times 10^3$, 30% higher than the deterministic equilibrium. Hence, although there is little change in the $s(K)$ curves, use of the optimal stochastic policy can effectively lead to a substantially larger stock of whales (on average), while decreasing the (mean) optimal capacity by only 11%, from $K = 2250$ to $K = 2000$.

Performance of Optimal and Suboptimal Policies

At this point, two fundamental questions need to be addressed: how sensitive is the value of the fishery to changes in the policy curves $s(K)$ and $h(S)$ away from their optimum positions and how well do the policy functions obtained as optimal for a deterministic environment compare with "true" optimal policies for the stochastic fishery?

The very nature of optimal controls suggests that small variations in the controls should have even smaller effects on the value function (Ludwig 1980). This indeed appears to be the case in the present model. Using the deterministic version, the optimal policy function $h(S)$ for the base case prawn fishery was perturbed first upwards and then downwards by 10%. The reduction in the value function was approximately 1.0% in both cases, a result in agreement with Ludwig's point that the variation in the value function should be proportional to the square of the fractional deviation in the policies.

It was shown above that policies that take into account fluctuations in the fishery's environment can differ from their deterministic counterparts by as much as 30–40%, for reasonable parameter combinations. Optimal fisheries investment, then, can be significantly higher, or significantly lower, under uncertainty. However, the optimal value function appears to be rather insensitive to changes in the policy functions away from their optimal positions. Table 1 gives the value function (at discrete grid points in the $R-K$ plane) based on the optimal policies for a stochastic (σ = 0.58) fishery with a = 14 and δ = \$0.0832 × 10^6. Table 2 represents comparable results using the optimal policies for the corresponding deterministic fishery, but evaluated in a stochastic (σ = 0.58) environment.

By comparing these value functions point by point, one can see that the loss from using the deterministic policy is never more than \$0.2 million. For example, with S = 2.1 × 10^6 (the quasi-equilibrium escapement) and K = 0, the optimal investment for the stochastic fishery is I^* = 12.5, while the deterministic policy produces I = 8.8, a 30% underinvestment. However, the reduction in value of the fishery caused by using the deterministic policy is roughly 3%, or only \$0.16 million, a rather negligible amount when one considers the overall lack of precision attainable in real-world fisheries. Hence, while effects of uncertainty on the policies themselves can be considerable, the use of "incorrect" deterministic policies may not reduce the fishery's value significantly. The implications of this result are discussed below.

Discussion

Results obtained here point to three primary conclusions, involving (i) the qualitative differences between deterministic and stochastic fisheries, (ii) the upside/downside determinants of optimal investment under uncertainty, and (iii) the performance of deterministic versus stochastic strategies. On the first point, heuristic analysis of the stochastic model indicated that optimal investment and escapement policies under uncertainty should not differ qualitatively from the deterministic case; numerical results confirmed this expectation. However, by simulating stochastic sample paths and steady-state distributions, it was shown that in practice the appearance of an optimally managed stochastic fishery is quite different from that of its deterministic counterpart. Even in the long run, optimal fleet capacity in a stochastic environment should be expected to fluctuate over a fairly wide range. This range will be greater the slower growing and the more variable the resource stock. In particular, an optimal investment program should allow the capital stock to respond positively to unusually "good" years, either by permitting increased entry of vessels or by direct acquisition of extra capital. This is done in full knowledge that idle capacity will then be greater in the "bad" years. (The possibility of various "political" pressures leading to the overutilization of this new capacity may be a real danger but has not been included in the model discussed here.)

TABLE 2. Expected value function $V(R,K)$ for a σ = 0.58 stochastic environment, but determined using the "optimal" policies for the corresponding deterministic fishery. Recruitment is given in millions of kilograms, capacity K in standardized vessels, and value in millions of Australian dollars.

	Recruitment							
Capacity	0.20	0.50	1.00	2.00	3.00	4.50	7.00	20.0
0.0	4.48	4.90	5.23	5.75	6.11	6.45	6.76	7.25
3.0	4.63	5.08	5.46	5.96	6.43	6.95	7.57	9.63
6.0	4.77	5.25	5.68	6.18	6.69	7.38	8.28	11.7
9.0	4.90	5.41	5.87	6.39	6.93	7.75	8.88	13.5
12.0	5.03	5.57	6.06	6.62	7.15	8.06	9.40	15.1
15.0	5.16	5.72	6.24	6.84	7.37	8.35	9.84	16.4
18.0	5.28	5.87	6.41	7.05	7.58	8.57	10.2	17.6
21.0	5.40	6.01	6.58	7.24	7.77	8.76	10.6	18.7

The balancing of upside and downside risks has been used here to explain the quantitative effects of uncertainty on optimal fleet capacity. In all fisheries investment decisions there exists an upside risk of foregone benefits in exceptionally good years and a downside risk of suffering idle capacity in bad years. Whether optimal investment will be higher or lower under uncertainty (compared with the deterministic optimum) depends on the relative importance of these risks, which in turn are influenced by fishery parameter values.

The intrinsic biomass growth rate (a) and the ratio δ/cT of unit capital cost to maximum yearly operating costs are of particular importance in this regard. Figure 9 shows (a,δ) combinations (with cT fixed) for which optimal capacity is generally higher (+) or lower (−) under uncertainty (σ = 0.58), together with a rough curve dividing the two regions.

In general, investment will be higher under uncertainty if the resource is fast-growing and capital is relatively cheap. In this case, the upside benefits of extra fleet capacity are substantial, while the downside risks of idle capacity are not as critical. The reverse will be true for a slow-growing stock with expensive capital. This suggests a guiding principle for estimating the qualitative effect of randomness, without undertaking a full stochastic analysis: if the ratio of unit capital cost to yearly operating costs seems fairly low, and if the resource is reasonably fast-growing (as with prawn stocks), then investment is likely to be at least somewhat higher under uncertainty. This information may be useful in determining whether a fishery has indeed experienced overinvestment, or whether apparent excess capacity is in fact optimal given the history of the fishery's development in the face of uncertain future stock sizes.

The framework of upside versus downside fisheries investment risks is useful as well in analyzing the effects of other model parameters. It was found in particular that lower depreciation rates and lower discount rates increase the

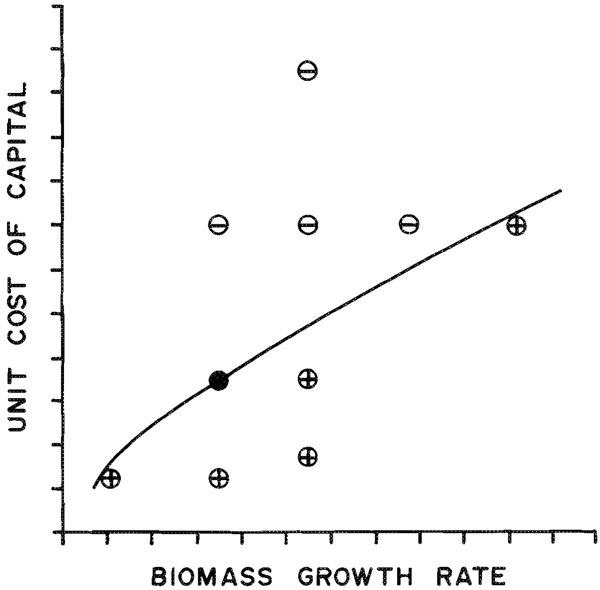

FIG. 9. Interaction between biomass growth rate and cost of capital in determining the role of uncertainty in fisheries investment. Points shown represent (a,δ) combinations that have been considered. Investment increases (+) or decreases (−) with uncertainty. (Diagram is not drawn to scale and the dividing line is approximate.)

tendency for optimal investment to be higher under uncertainty. In addition, the general results discussed above were found to apply to the case of Ricker stock-recruitment and to the aggregated whaling fishery.

Examination of the quantitative results obtained here shows that for moderate levels of variability, and reasonable parameter combinations, the relative difference between stochastic and deterministic optimal fleet capacities can reach 30−40%. This results in substantial over- or under-investment in fleet capacity when the deterministic model is used in place of a full stochastic model. Target escapements, on the other hand, tended to be remarkably insensitive to the level of uncertainty in the fishery, a result in agreement with previous research.

Irreversibility of investment increases the importance of inherent uncertainty in the fishery. This is particularly the case for fisheries with slow-growing resource stocks, where the occurrence of an unusually "bad" year may lead to capital lying idle for a substantial part of its economic life. However, in accordance with the work of other researchers (e.g. Lewis 1981), results obtained here show that for the linear-cost risk-neutral fishery model, optimal policies recognizing the stochastic nature of the fishery tended to perform only somewhat better than policies based on the corresponding deterministic model. In other words, the use of a deterministic model was sufficient in most cases to produce policies with near-optimal performance (on average). Indeed, it is apparent that any investment strategy sufficiently near the optimal will perform almost optimally. While this result certainly does not reduce the importance of uncertainty to fisheries management, it does imply that with linear costs and risk neutrality, economic optimization is "forgiving"; other objectives (conservation, job creation, etc.) can be pursued with little loss in the fishery's economic value, as long as the modified policy remains near the optimal strategy (with a deviation of roughly ±20% being reasonable).

Lewis (1981) has shown that this "forgiving" nature need not apply when either nonlinear costs or risk aversion are included. Since results obtained here show that investment policies can be strongly affected by uncertainty, even with linear costs and risk neutrality, the incorporation of additional nonlinearities may make the use of stochastic rather than deterministic investment policies particularly important to the fishery's performance. This will be a topic of future research.

This paper has emphasized the determination of optimal investment policies in the face of uncertainty arising through stochastic fluctuations in the resource stock. It has been assumed that the underlying population dynamics, represented by the stock-recruitment function $F(S)$, are given. However, in practice, stock-recruitment parameters (and some economic data) are known only imprecisely. In such circumstances, parameter estimates must be refined from year to year as new information becomes available. The effects of this parameter uncertainty on fisheries investment, and the role of parameter updating in overcoming these uncertainties, are considered in Charles (1983b). It is found there that initial errors in stock-recruitment parameter estimates can lead to considerable long-term overcapacity. Parameter uncertainty, it seems, plays an important role alongside stochastic variability in determining optimal investment strategies. The formulation of adaptive management policies, in which fisheries investment responds to new information, and is used in turn as a tool to acquire information, promises to be a fruitful area for further research.

Acknowledgments

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