

A Bioeconomic Analysis of Sequential Fisheries: Competition, Coexistence, and Optimal Harvest Allocation Between Inshore and Offshore Fleets

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A bioeconomic model is developed to determine optimal harvest allocation between "offshore" and "inshore" fleets exploiting a single fish stock in sequential fisheries. The socially optimal policy for maximizing total discounted rent is determined in terms of optimal escapement levels in each fishery. Whether exclusion or coexistence of the two fleets occurs under open access and under optimal management is found to depend primarily on inshore/offshore price and cost ratios, together with biological parameters related to the age structure of the fish stock. We discuss how fishery regulations, such as separate landings taxes imposed on each fleet, can be used to jointly optimize open-access exploitation in sequential fisheries.

Les auteurs ont élaboré un modèle bioéconomique pour déterminer la répartition optimale des contingents entre les secteurs côtier et hauturier qui exploitent un seul stock de poissons au cours de pêches consécutives. La politique sociale optimale permettant de maximiser la rente totale escomptée est déterminée en fonction des niveaux optimaux d'échappée dans chaque pêche. L'exclusion ou la coexistence des deux flottilles en vertu d'un accès libre et d'une gestion optimale dépend surtout des rapports coûts / avantages de la pêche côtière et hauturière et des paramètres biologiques de la structure des âges dans les stocks de poisson. Les auteurs examinent comment des règlements de pêche, comme l'imposition à chaque flottille de taxes au débarquement de montants distincts, peuvent être utilisés pour optimiser conjointement l'exploitation à accès libre de pêches consécutives.

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While the biological complexities of fisheries management have received considerable attention in the literature (for example, Clark 1976; Ludwig and Walters 1982; Mendelsohn 1978; Reed 1974), an equally important complicating aspect involves complexity in the structure of the fishery itself. In many fisheries, a single fish population is harvested sequentially by a series of separate, discrete fisheries. The pattern of such "gauntlet" fisheries can include (i) harvesting by different fleets at different stages in the life cycle of the fish stock, (ii) simultaneous or near simultaneous harvesting by different fleets, or (iii) sequential harvesting by the same fleet but at two separate times and possibly at different geographical locations. Typically, these fisheries involve conflicts between users of the fish resource, due to differences in types of fishing gear, location, or timing of fishing.

In cases such as these, management of each fishery in isolation ignores the fact that the fisheries are linked through their exploitation of the same fish stock, while aggregating the fisheries together ignores economic differences between them, as well as differences in the biological impact of each fishery

on the fish stock. The optimal fleet mix and optimal catch allocation to each fishery must involve a balance between these economic and biological factors. A model to accomplish this is developed herein, to provide an analytical framework for examining optimal harvesting strategy, optimal catch allocation, and questions of exclusion or coexistence of the fisheries under optimal management and under open-access conditions.

In the model, it will be assumed, for simplicity, that precisely two fisheries are possible each season; the first (referred to here as the *offshore* fishery) is based on the entire fish stock, while the second (the *inshore* fishery) harvests only spawners. Note that if the entire vulnerable offshore population moves inshore to spawn each year, then both fisheries are in fact harvesting from the entire fish stock; an example of this special case will be discussed in the following section.

The model is applied to a study of economic and policy aspects concerning the allocation question. How do economic factors (unit prices and costs) determine the equilibrium configuration of the fishery: exclusion of one of the fisheries or coexistence of both? How does the open-access bioeconomic equilibrium compare with optimal harvest allocation? How can

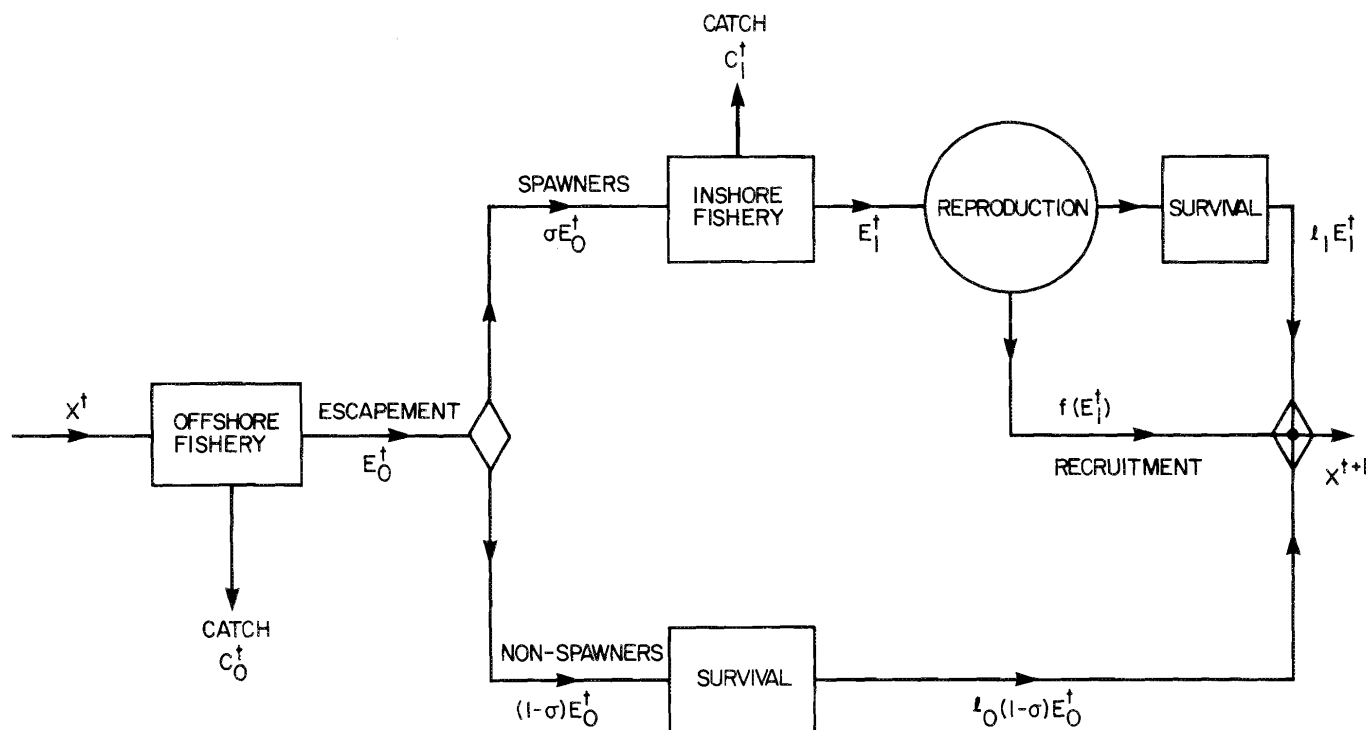


FIG. 1. General structure of a sequential fishery, as assumed in the model. Note that if $\sigma = 1$, implicit age structure is suppressed, and the model assumes two successive fisheries on the same fish stock.

regulatory tools be used to move the open-access fishery towards optimum?

The optimal management of sequential fisheries has received little attention to date, at least within the context of bioeconomic analysis. The related question of multiple fleets harvesting a single fish stock has received some attention, notably in the work of Clark and Kirkwood (1979) and McKelvey (1983). These papers examined problems of optimal fleet mix from the perspective of determining optimal capacities for each fleet. However, by assuming that the fleets harvest simultaneously, the structural aspects of sequential fisheries were not emphasized.

The Model

To motivate the derivation of a suitable model, we first discuss a simplified depiction of a particular gauntlet fishery, targeted on British Columbia chinook salmon (*Oncorhynchus tshawytscha*). Salmon trollers harvest both spawners (adults) and nonspawners (juveniles), while net boats (gill-netters and purse-seiners) tend to catch primarily spawning chinook. The average age and weight of a troll-caught salmon is typically less than that of a net-caught (spawning) fish, but since the former are considered to be of higher quality than those caught by inshore nets closer to the spawning grounds, the price per fish is assumed to be higher for chinook salmon caught by trollers. However, fishing costs per unit harvest will also be higher for trollers in general, since greater distances must be travelled to reach offshore fishing grounds. Finally, while a fish foregone by the troll fleet may contribute neither to the inshore net catch nor to the spawning effort (due to natural mortality), a fish that escapes from inshore nets has a relatively high probability of spawning and hence is of greater benefit to future chinook stock sizes. These factors combine to make management of such fisheries a particularly complex problem.

Consider now an aggregated fish stock whose size (in numbers of fish) at the beginning of the fishing season in year t is X^t . Suppose that the offshore fishery acts first on the stock and removes C_0^t fish, with a resulting escapement of E_0^t . Neglecting natural mortality during the offshore fishing season, we have $E_0^t = X^t - C_0^t$. Suppose that a fixed proportion σ of the escapement from the offshore fishery move inshore to spawn in each year, and that the inshore fishery acts only on this spawning group. If the inshore catch is denoted by C_1^t , then $E_1^t = \sigma E_0^t - C_1^t$ is the inshore escapement. (Unless otherwise indicated the superscript will refer to time and the subscripts 0 and 1 to offshore or inshore, respectively.) Suppose that this spawning stock of E_1^t fish results in a subsequent recruitment $f(E_1^t)$ to the stock in year $t + 1$. Assume further that a proportion ℓ_1 of the spawners in year t survive to be part of the stock in year $t + 1$, and that a proportion ℓ_0 of the nonspawners similarly survive. The dynamics of the model are then given by the equation

$$(1) \quad X^{t+1} = \ell_0(1 - \sigma)E_0^t + \ell_1 E_1^t + f(E_1^t)$$

where

$$E_0^t = X^t - C_0^t$$

and

$$E_1^t = \sigma E_0^t - C_1^t.$$

The assumed fishery structure is depicted in Fig. 1.

For convenience we will lump the terms involving surviving spawners and new recruits into a single term by letting

$$(2) \quad F(E_1^t) = \ell_1 E_1^t + f(E_1^t).$$

This function, F , then represents an overall spawner to future-stock relationship, and will be assumed to be concave and increasing (exhibiting "normal compensation"). Equation (1) can then be written:

$$(3) \quad X^{t+1} = \beta E_0^t + F(E_1^t)$$

where $\beta = \ell_0(1 - \sigma)$. This is the basic dynamic equation, subject to the natural constraints $0 \leq E_0^t \leq X^t$ and $0 \leq E_1^t \leq \sigma E_0^t$. This model in its generality applies to many exploited fish stocks. Specific cases are discussed at the end of this section.

We now turn to the economic aspects of the model. Let p_0, p_1 denote the price per fish (landed or wholesale) for the offshore and inshore fisheries, respectively. We assume in general that $p_0 > p_1$. Suppose that for both fisheries the assumptions underlying the Schaefer production function hold (see Clark 1976, p. 235). In this case the cost of catching a fish in the offshore fishery when the target population is at level x is c_0/x . Similarly in the inshore fishery the corresponding marginal cost is c_1/x . The total costs of the two harvests in year t are thus $c_0 \log(X^t/E_0^t)$ and $c_1 \log(\sigma E_0^t/E_1^t)$, obtained by integrating the marginal costs between initial and final stock sizes for each fishery. We shall assume that inshore fishing is cheaper per unit effort than offshore fishing, i.e. that $c_0 > c_1$.

The annual rent produced by the two fisheries is thus

$$(4) \quad \pi^t = p_0(X^t - E_0^t) - c_0 \log(X^t/E_0^t) + p_1(\sigma E_0^t - E_1^t) - c_1 \log(\sigma E_0^t/E_1^t).$$

Some special cases of the model are of interest.

"Chinook Salmon" Model

Setting $\ell_1 = 0$ in the model is equivalent to assuming that all spawners die after spawning, a biological feature characteristic of all species of Pacific salmon. With $\sigma < 1$ the model assumes that fish reach spawning maturity at several different ages, as is the case with chinook salmon. The offshore and inshore fisheries in this case correspond to troll and net fleets, as discussed previously.

"Pink Salmon" Model

If $\ell_1 = 0$ and $\sigma = 1$ we have a model in which all fish spawn at the same age and die after spawning. This is the case for pink salmon (*Oncorhynchus gorbuscha*) in the Northeast Pacific, which invariably spawn at age 2. Again the two relevant fisheries are the troll fishery (offshore) and the net fishery (inshore). In a given fishing season, the troll fishery takes place first, with the net fishery then acting on the escapement from the troll fishery.

From the point of view of analysis the pink salmon model is considerably simpler than the general model, since with $\beta = 0$ in equation (3) we have $X_{t+1} = F(E_1^t)$. The chinook model, on the other hand, does not provide any real simplification.

"Herring" Model

In recent history, the British Columbia Pacific herring (*Clupea harengus pallasi*) fishery has consisted of three primary components. Currently, the first fishery each season serves the relatively small, nonlucrative "food and bait" market and is aimed primarily at mature fish, with a limited harvest of juvenile nonspawners. Later in the season, after mature stocks of herring have moved to their spawning grounds, a "roe fishery" takes place, in which the spawning stock is harvested for the Japanese herring roe market.

This latter fishery began in the early 1970s; before that time the primary component to the British Columbia herring fishery was a reduction fishery producing fish meal and fish oil. This

type of fishery, which no longer exists in British Columbia but is common in many locations around the world, harvests from the entire population (spawners and juveniles alike) in a relatively nonselective manner.

Since individual herring require differing lengths of time to reach sexual maturity, but then are able to spawn for several successive years, our model can be used to address optimal management of a sequential herring fishery by setting $\ell_0 > 0$, $\ell_1 > 0$, and $0 < \sigma < 1$. However, since the "food and bait" fishery does not target equally on spawners and nonspawners, the model would require slight modification before it could be fully applied to this herring fishery.

Bionomic Equilibrium

In this section we consider the Gordon (1954) bionomic equilibrium, resulting from open access to both inshore and offshore fisheries.

For the offshore fishery, when the fish population is at level x , the marginal return (per extra fish caught) is

$$p_0 - c_0/x.$$

Thus there will be an offshore fishery only if the initial population level x exceeds c_0/p_0 . If this is the case then in the open-access situation with no limitations on effort, the offshore fishery will harvest the stock down to c_0/p_0 , the zero marginal rent level.

Similarly the inshore fishery will operate only if the initial target population level σE_0 exceeds c_1/p_1 where E_0 is the escapement from the offshore fishery, $E_0 = \min(x, c_0/p_0)$. In this case with open access the resulting escapement level will be c_1/p_1 , the zero marginal rent level.

Thus the bionomic equilibrium escapement levels \hat{E}_0 and \hat{E}_1 and initial stock level \hat{X} are given by the three equations

$$(5) \quad \hat{E}_0 = \begin{cases} \hat{X} & \text{if } \hat{X} \leq c_0/p_0 \\ c_0/p_0 & \text{if } \hat{X} > c_0/p_0 \end{cases}$$

$$(6) \quad \hat{E}_1 = \begin{cases} \sigma \hat{E}_0 & \text{if } \sigma \hat{E}_0 \leq c_1/p_1 \\ c_1/p_1 & \text{if } \sigma \hat{E}_0 > c_1/p_1 \end{cases}$$

$$(7) \quad \hat{X} = \beta \hat{E}_0 + F(\hat{E}_1).$$

The variable \hat{X} can be eliminated from (5) and (7) through the use of some mathematical manipulation to show that

$$\hat{X} = \beta \hat{E}_0 + F(E_1) > c_0/p_0 \quad \text{if and only if } (1 - \beta)^{-1}F(\hat{E}_1) > c_0/p_0.$$

This allows the replacement of (5), (6), and (7) with the two equations

$$(8) \quad \hat{E}_0 = \begin{cases} (1 - \beta)^{-1}F(\hat{E}_1) & \text{if } (1 - \beta)^{-1}F(\hat{E}_1) \leq c_0/p_0 \\ c_0/p_0 & \text{if } (1 - \beta)^{-1}F(\hat{E}_1) > c_0/p_0 \end{cases}$$

$$(9) \quad \hat{E}_1 = \begin{cases} \sigma \hat{E}_0 & \text{if } \sigma \hat{E}_0 \leq c_1/p_1 \\ c_1/p_1 & \text{if } \sigma \hat{E}_0 > c_1/p_1, \end{cases}$$

i.e. $\hat{E}_0 = \min\{(1 - \beta)^{-1}F(\hat{E}_1), c_0/p_0\}$ and $\hat{E}_1 = \min\{\sigma \hat{E}_0, c_1/p_1\}$.

The solution to this pair can be obtained graphically. Four cases arise depending on which of the four regions in the E_0 - E_1 plane defined by the curves $E_0 = (1 - \beta)^{-1}F(E_1)$ and $E_1 = \sigma E_0$, the point $P = (c_0/p_0, c_1/p_1)$ falls in (see Fig. 2).

Case 1

If the point $P = (c_0/p_0, c_1/p_1)$ falls in region A, i.e. if c_1/p_1

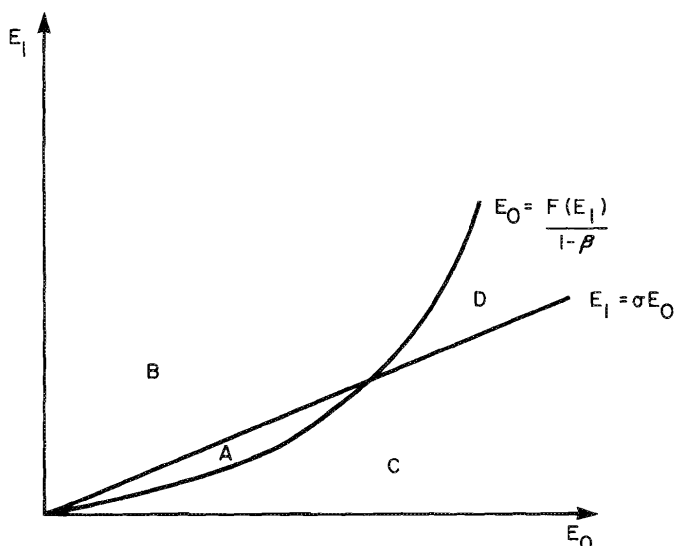


FIG. 2. The qualitative nature of the open-access bionomic equilibrium is determined by the location of the point $(c_0/p_0, c_1/p_1)$, representing offshore and inshore cost/price ratios, in relation to the four regions of the E_0 - E_1 space.

$< \sigma c_0/p_0$ and $c_0/p_0 < (1 - \beta)^{-1} F(c_1/p_1)$, then the graphs of equations (8) and (9) are given by the bold curves in Fig. 3a, and their solution is given by the point of intersection P . In this case in bionomic equilibrium both inshore and offshore fisheries coexist, with the offshore catch being $\beta c_0/p_0 + F(c_1/p_1) - c_0/p_0$ and the inshore catch being $\sigma c_0/p_0 - c_1/p_1$.

Case 2

If the point P falls in region B, then Fig. 3b applies, and the solution is at the point Q , where the bionomic equilibrium escapements are $\hat{E}_0 = c_0/p_0$ and $\hat{E}_1 = \sigma c_0/p_0$. In this case the inshore fishery is excluded in bionomic equilibrium.

Case 3

If the point P falls in region C, the solution lies at the point Q in Fig. 3c, so that the bionomic equilibrium escapements are $\hat{E}_1 = c_1/p_1$ and $\hat{E}_0 = (1 - \beta)^{-1} F(c_1/p_1)$. In this case the offshore fishery is excluded in bionomic equilibrium.

Case 4

If the point P falls in region D, then the solution to (8) and (9) is given by the intersection, Q , of the bold curves in Fig. 3d. In this case, \hat{E}_0 is the solution to $\hat{E}_0 = (1 - \beta)^{-1} F(\sigma \hat{E}_0)$, and $\hat{E}_1 = \sigma \hat{E}_0$, so that \hat{E}_0 is the unfished equilibrium stock level and \hat{E}_1 is the unfished equilibrium spawning level. In this case neither fishery can make a profit.

The results above can be summarized in terms of cost/price ratios. If there is open access to both fisheries then there will be an inshore fishery present in bionomic equilibrium if and only if $c_1/p_1 < \sigma c_0/p_0$, and there will be an offshore fishery present in bionomic equilibrium if and only if $c_0/p_0 < (1 - \beta)^{-1} F(c_1/p_1)$. Coexistence of both fisheries in bionomic equilibrium will be possible if and only if

$$\frac{1}{\sigma} \left(\frac{c_1}{p_1} \right) < \left(\frac{c_0}{p_0} \right) < \frac{1}{1 - \beta} F \left(\frac{c_1}{p_1} \right),$$

a condition that states that neither cost/price ratio can be too high with respect to the other.

Dynamic Optimization

In this section the optimal policy to maximize the discounted stream of rents from the resource will be analyzed in a dynamic setting.

It will be assumed that fishery rents π^t earned in year t will be discounted by a factor¹ α^t to a "year-zero" or present value $\alpha^t \pi^t$. The discount factor α is related to the annual discount rate, i , by $\alpha = 1/(1 + i)$.

For a given exploitation pattern characterized by the sequence of escapements $\{(E_0^t, E_1^t)\}_{t=0}^{\infty}$, the present value of the stream of rents is

$$(10) \quad J = \sum_{t=0}^{\infty} \alpha^t \pi^t$$

where π^t is given by (4). The overall social objective will be assumed to be the maximization of the present value J over the sequence $\{(E_0^t, E_1^t)\}_{t=0}^{\infty}$ subject to the constraints

$$(11) \quad 0 \leq E_0^t \leq X^t$$

$$(12) \quad 0 \leq E_1^t \leq \sigma E_0^t$$

and the dynamic equation

$$(13) \quad X^{t+1} = \beta E_0^t + F(E_1^t).$$

This is a control problem involving a single state variable (X^t) and two control variables (E_0^t and E_1^t). The solution will be studied by means of the Discrete Maximum Principle (e.g. see Cannon et al. 1970).

To apply the Discrete Maximum Principle we define a Hamiltonian H^t :

$$(14) \quad H^t = \alpha^t \pi^t + \lambda^t (X^{t+1} - X^t)$$

where π^t is given by (4) and λ^t is the adjoint or co-state variable, representing the shadow value of the fish stock. Writing out the Hamiltonian in full, we obtain

$$(15) \quad H^t = \alpha^t [p_0 (X^t - E_0^t) + p_1 (\sigma E_0^t - E_1^t) + c_0 \log (E_0^t/X^t) + c_1 \log (E_1^t/\sigma E_0^t)] + \lambda^t [\beta E_0^t + F(E_1^t) - X^t].$$

In simple applications of the Discrete Maximum Principle, this Hamiltonian would be maximized over the control variables, and the resulting optimality equations combined with the adjoint equation for λ , to produce the overall solution. In this case the analysis is not as straightforward, due to the complexities involved in dealing with two controls and one state variable. The Hamiltonian must be considered as a function of the state X , treating the controls E_0 and E_1 as parameters temporarily. If $X > E_0$, equation (15) holds without modification. However, if instead we have $X \leq E_0$, which allows both for the case of no offshore fishery ($X = E_0$) and for mathematical continuity in the state variable X to levels less than E_0 , then one must rewrite the Hamiltonian, replacing E_0^t by X^t throughout. In other words:

$$(16) \quad H^t = \alpha^t [p_1 (\sigma X^t - E_1^t) + c_1 \log (E_1^t/\sigma X^t)] + \lambda^t [(\beta - 1)X^t + F(E_1^t)] \quad \text{if } X^t \leq E_0^t.$$

This essentially states that, for a fixed target offshore escapement E_0^t , if $X^t \leq E_0^t$, then there will be no offshore fishery and hence no offshore revenues or costs.

The Hamiltonian must be considered, therefore, as being piecewise defined by (15) and (16). Proceeding now with the

¹In all of the following, α^t represents α raised to the t th power.

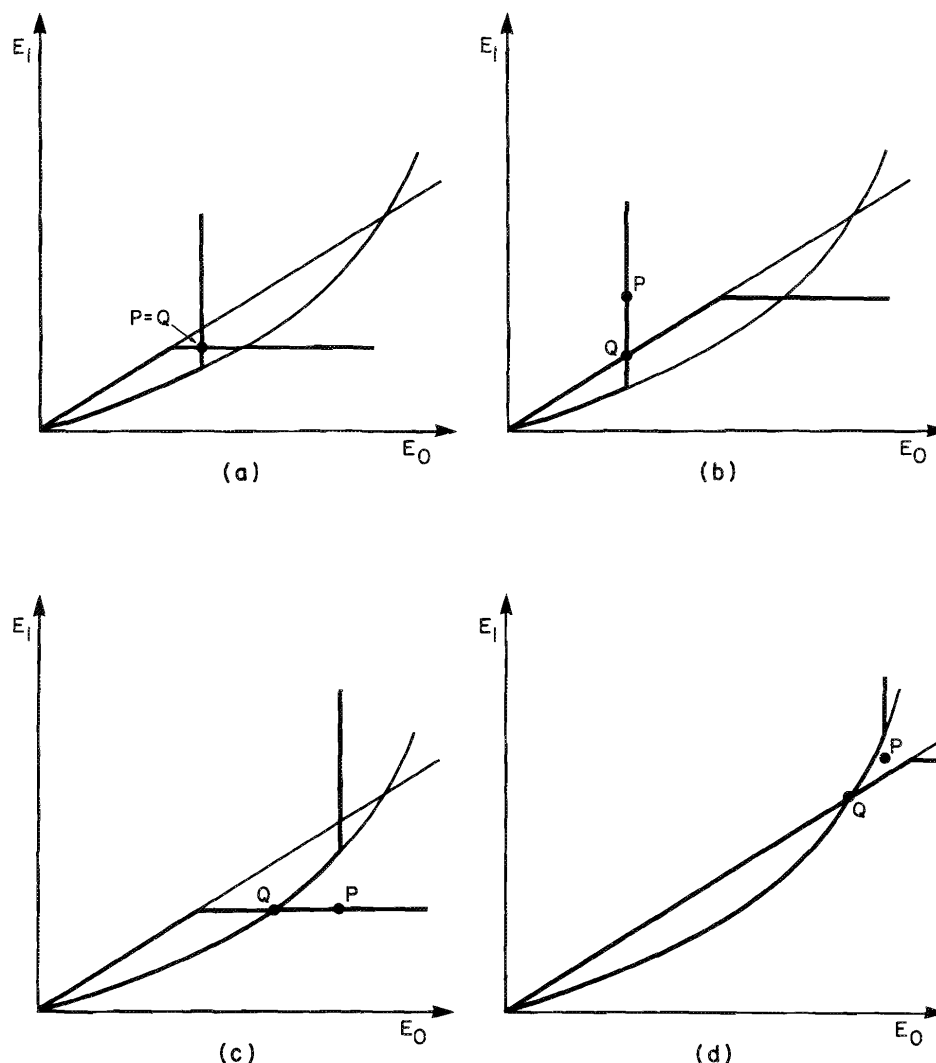


FIG. 3. Coexistence and exclusion in bionomic equilibrium. The point $(c_0/p_0, c_1/p_1)$ in E_0 - E_1 space is represented by P . The bionomic equilibrium escapement pair (\hat{E}_0, \hat{E}_1) is represented by Q , the intersection of the bold lines. (a) Coexistence of the two fleets; (b) exclusion of the inshore fishery; (c) exclusion of the offshore fishery; (d) the case in which neither fishery can make a profit, so that both are excluded.

derivation of the adjoint equation, we have

$$\lambda^t - \lambda^{t-1} = -\frac{\partial H^t}{\partial X^t} = \begin{cases} -\alpha^t(p_0 - c_0/X^t) + \lambda^t & \text{if } X^t > E_0^t \\ -\alpha^t(\sigma p_1 - c_1/X^t) + (1 - \beta)\lambda^t & \text{if } X^t \leq E_0^t. \end{cases}$$

This can be simplified to the following:

$$(17) \quad \lambda^t = \begin{cases} \alpha^{t+1}(p_0 - c_0/X^{t+1}) & \text{if } X^{t+1} > E_0^{t+1} \\ \alpha^{t+1}\sigma(p_1 - c_1/\sigma X^{t+1}) + \beta\lambda^{t+1} & \text{if } X^{t+1} \leq E_0^{t+1}. \end{cases}$$

Equation (17) has a straightforward but interesting interpretation. If next year's fish stock, X^{t+1} , is sufficiently large so that an offshore fishery will be desirable ($X^{t+1} > E_0^{t+1}$), then the marginal shadow value of a nonexploited fish this year, λ^t , is equal to the marginal net benefits that will accrue next year, $p_0 - c_0/X^{t+1}$, discounted to year zero. On the other hand, if no offshore fishery will occur next year ($X^{t+1} \leq E_0^{t+1}$), then the shadow value of the resource is given by the sum of two components: (a) the discounted marginal net benefits of the

spawning fraction of the fish stock, in next season's inshore fishery, and (b) the shadow value of the nonspawning portion of next year's fish stock that survives to the following year.

In theory, equations (15), (16), and (17) can be analyzed to obtain the optimal dynamic solution. However, the piecewise definition of the Hamiltonian and the adjoint variable greatly complicates the analysis, which will not be undertaken further in this paper. Instead, we turn now to the question of optimal equilibrium allocation, which can be analyzed in a more complete manner.

Optimal Equilibrium

In this section, equations are derived for the optimal stock and escapement levels in equilibrium. These can be related to the bionomic equilibrium under open access, in order to determine regulatory schemes that will produce a socially optimal pattern of exploitation in both the inshore and offshore fisheries under open-access conditions.

Let X , E_0 , and E_1 now represent steady-state stock levels

satisfying $X = \beta E_0 + F(E_1)$. Inserting these quantities in the expressions for the Hamiltonian H' and the adjoint variable λ' , we have

$$(18) \quad H' = \begin{cases} \alpha'(-p_0 E_0 + c_0 \log E_0 + \sigma p_1 E_0 - c_1 \log E_0) \\ \quad + \beta \lambda' E_0 + \alpha'(-p_1 E_1 + c_1 \log E_1) \\ \quad + \lambda' F(E_1) + \alpha'(p_0 X - c_0 \log X) - \lambda' X & \text{if } X > E_0 \\ \alpha'(\sigma p_1 X - p_1 E_1 - c_1 \log(\sigma X) + c_1 \log E_1) \\ \quad + \lambda' \beta X + \lambda' F(E_1) - \lambda' X & \text{if } X \leq E_0 \end{cases}$$

$$(19) \quad \lambda' = \begin{cases} \alpha'^{t+1}(p_0 - c_0/X) & \text{if } X > E_0 \\ \alpha'^{t+1}(\sigma(p_1 - c_1/\sigma X) + \beta \lambda'^{t+1}) & \text{if } X \leq E_0. \end{cases}$$

It is necessary to analyse the three possible outcomes (inshore fishery only, offshore fishery only, and coexistence) separately, to allow for the piecewise definition of H and λ .

Coexistence

If coexistence is optimal, we must have $X > E_0$ (for an offshore fishery to occur) so that the adjoint variable λ' is specified from (19). However, it is also true in such a case that $\partial H/\partial E_1 = 0$, so that

$$\lambda' = \alpha' \frac{p_1 - c_1/E_1}{F'(E_1)}.$$

Combining the two expressions for λ' , we obtain

$$(20) \quad p_1 - \frac{c_1}{E_1} = \alpha F'(E_1) \left(p_0 - \frac{c_0}{X} \right)$$

where $X = \beta E_0 + F(E_1)$. One must now consider in addition the decision variable E_0 ; setting $\partial H/\partial E_0 = 0$ produces

$$(21) \quad p_0 - \frac{c_0}{E_0} = \sigma \left(p_1 - \frac{c_1}{\sigma E_0} \right) + \alpha \beta \left(p_0 - \frac{c_0}{X} \right).$$

With $X = \beta E_0 + F(E_1)$, equations (20) and (21) can be solved simultaneously for both optimal escapement levels. These equations can be interpreted as follows. Given an offshore escapement E_0 , equation (20) states that the optimal inshore escapement E_1 should be such that immediate marginal benefits $p_1 = c_1/E_1$ are balanced by the discounted future marginal rents that are foregone. Equation (21) sets the offshore escapement, given E_1 , at a level such that immediate return from an additional unit harvest is equated to the sum of (a) the resulting foregone benefit to this season's inshore fishery and (b) the discounted marginal loss to next year's offshore fishery (due to harvest of nonspawners this season). Note that equations (20) and (21) can be written, respectively, as

$$E_1 = \phi_1(E_0)$$

and

$$E_0 = \phi_0(E_1)$$

where the function ϕ_1 is defined by equating the $(p_0 - c_0/X)$ terms in (20) and (21) to obtain

$$(20') \quad \frac{p_1 - c_1/E_1}{F'(E_1)} = \frac{1}{\beta} \left[(p_0 - \sigma p_1) - \frac{c_0 - c_1}{E_0} \right]$$

while $E_0 = \phi(E_1)$ can be formulated from (21) simply as the

positive solution of the quadratic equation

$$(21') \quad E_0^2 + B E_0 + C = 0$$

where B and C are linear functions of $F(E_1)$.

Straightforward mathematical analysis shows that the functions ϕ_1 and ϕ_0 can be portrayed qualitatively as in Fig. 4a, with the optimal equilibrium given by the intersection point p .

Also shown in Fig. 4a are the constraint curves $E_1 = \sigma E_0$ and $E_0 = (1 - \beta)^{-1} F(E_1)$. As fishery characteristics vary, the coexisting equilibrium solution given by the simultaneous solution of (20) and (21) approaches one of these constraints in a smooth manner. In other words, it can be shown that the following single-fishery optimality equations (offshore only, inshore only) are identical (modulo some simple algebra) to the coexisting solution subject to the additional constraint $E_1 = \sigma E_0$ or $E_0 = X$, respectively. If the intersection of the two optimality equations lies outside region A of Fig. 2, exclusion of one or both fisheries must be optimal.

Offshore Only

In this case, $X = \beta E_0 + F(\sigma E_0) > E_0$, since $E_1 = \sigma E_0$ and the inequality $X > E_0$ is necessary for an offshore fishery to occur. Hence, λ' is given by (19) with $X > E_0$, while

$$\frac{\partial H}{\partial E_0} = \alpha' \left(-p_0 + \frac{c_0}{E_0} \right) + \beta \lambda' + \lambda' \sigma F'(\sigma E_0) = 0$$

must hold for an interior optimum. Combining these expressions, we obtain the optimality equation for offshore escapement E_0 :

$$(22) \quad p_0 - \frac{c_0}{E_0} = \alpha [\beta + \sigma F'(\sigma E_0)] \left(p_0 - \frac{c_0}{X} \right).$$

The left-hand side of this equation is the marginal benefit obtainable from an additional unit harvest in the current season. The right-hand side is the marginal opportunity cost of that additional catch, in terms of foregone future harvest. It is the product of two terms: $\alpha(p_0 - c_0/X)$ is the discounted marginal value of an extra unit of the resource in the next season, while $\beta + \sigma F'(\sigma E_0)$ is a multiplier used to convert one offshore fish in the current year into the correct number of fish next year. This multiplier, in turn, is composed of two terms: survival of nonspawners plus incremental recruitment of "new" fish. Note that (22) can be rewritten in the standard Modified Golden Rule (MGR) form:

$$(23) \quad [\beta + \sigma F'(\sigma E_0)] \frac{p_0 - c_0/X}{p_0 - c_0/E_0} = \frac{1}{\alpha}.$$

In this form, it is evident that if $\beta = 0$ (no resident nonspawning stock offshore), then this "offshore only" result reduces to the standard single-stock, single-fishery solution (Clark 1976).

Figure 4b shows the situation diagrammatically. The optimality curves $E_1 = \phi(E_0)$ and $E_0 = \phi_0(E_1)$ intersect outside the constraint set and above the line $E_1 = \sigma E_0$. The desired equilibrium $(E_0^*, \sigma E_0^*)$, with E_0^* given by (22) or (23), will then lie on the line $E_1 = \sigma E_0$ at some point Q .

Inshore Only

In this case, $E_0 = X$ so $X = \beta X + F(E_1)$ and we thus have $X = (1 - \beta)^{-1} F(E_1)$. The adjoint variable is then given by (19) with $X \leq E_0$. An additional relationship can be obtained by

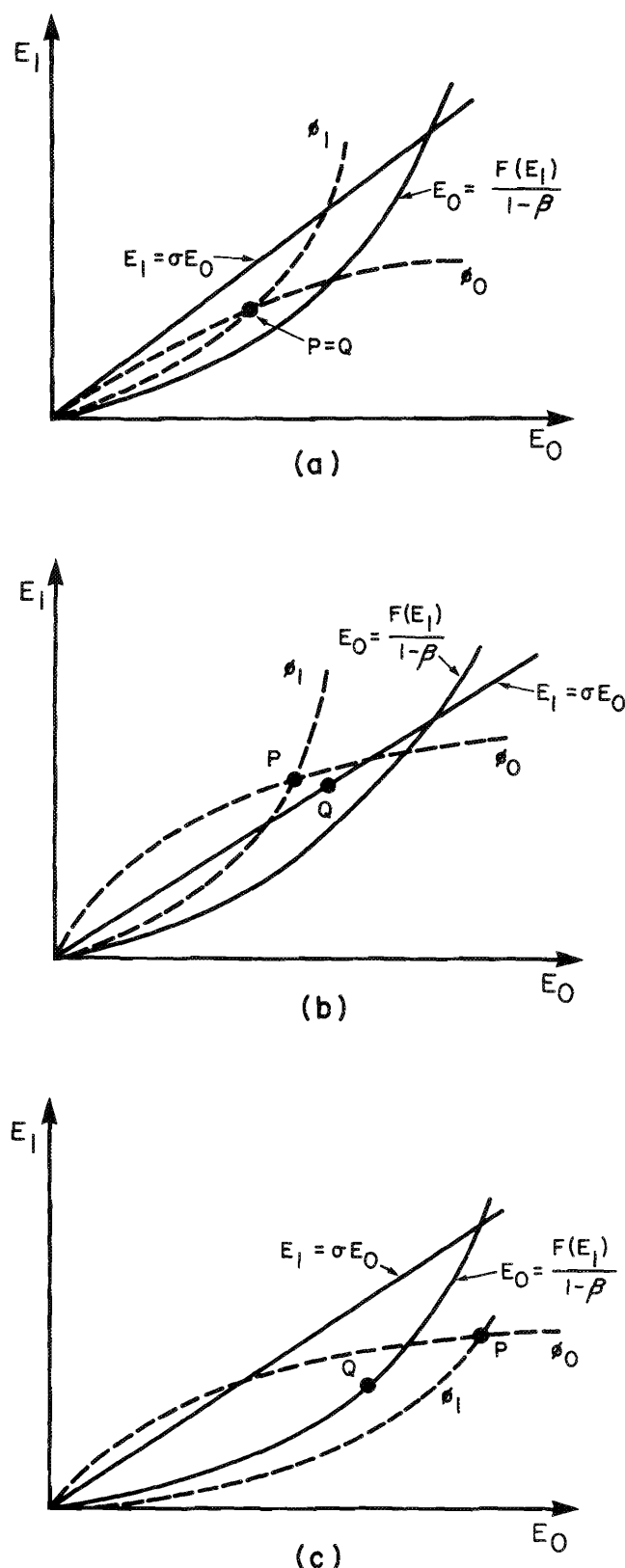


FIG. 4. Optimality curves $E_1 = \phi_1(E_0)$ and $E_0 = \phi_0(E_1)$ depicted (broken lines) together with the constraint curves $E_1 = \sigma E_0$ and $E_0 = F(E_1)/(1 - \beta)$. The location of the intersection point P of the optimality curves in relation to the constraints determines the nature of the optimal equilibrium. (a) Coexistence; (b) exclusion of the inshore fishery; (c) exclusion of the offshore fishery. Point Q represents the approximate location of the actual optimal equilibrium.

maximizing the Hamiltonian over inshore escapements E_1 :

$$\frac{\partial H}{\partial E_1} = \alpha' \left(-p_1 + \frac{c_1}{E_1} \right) + \lambda F'(E_1) = 0,$$

which can be rewritten

$$(24) \quad \lambda' = \alpha' \frac{p_1 - c_1/E_1}{F'(E_1)}.$$

Combining (24) with (19) and simplifying produces the optimality equation:

$$(25) \quad p_1 - \frac{c_1}{E_1} = \alpha \sigma F'(E_1) \left(p - \frac{c_1}{\sigma X} \right) + \alpha \beta \left(p_1 - \frac{c_1}{E_1} \right).$$

The left-hand side represents the immediate benefit of an incremental harvest in the inshore fishery. The right-hand side is the total foregone rents if that incremental catch is taken in the current season. This is composed of (a) the discounted loss to next year's inshore fishery and (b) the future loss due to foregone survival of offshore fish beyond the next season. Note that the term $(p_1 - c_1/E_1)$ appears on the right-hand side of (25), representing the current value of an additional unit of fish in the sea *next year*. Equation (25) can be written in a more standard MGR form:

$$(26) \quad \sigma F'(E_1) \frac{p_1 - c_1/\sigma X}{p - c_1/E_1} = \frac{1}{\alpha} - \beta = 1 + i - \beta$$

where i is the discount rate, $i = (1 - \alpha)/\alpha$.

Apart from the parameters β and σ , this represents the usual single-species optimality result. However, the role of non-spawning offshore fish becomes apparent from this formulation. The lower the fraction (σ) of offshore fish that spawn each year, the greater is the parameter β , since $\beta = \ell_0(1 - \sigma)$. In determining the optimal inshore escapement level, the offshore nonspawning stock plays the role of a "reserve," acting as a "bank account," on which the fishery draws each year to sustain

TABLE 1. Optimal equilibrium configuration (X , E_0^* , E_1^*) for each pair (p_1/p_0 , c_1/c_0) of price and cost ratios. Also shown is an indication of whether the equilibrium of the fishery involves only "inshore" fishing, only "offshore," or "coexistence." See text for details.

Price ratio p_1/p_0	Cost ratio c_1/c_0			
	0.1	0.3	0.5	0.7
1.0	4.08	4.58	4.88	4.92
	4.08	4.58	4.38	3.02
	1.60	1.94	2.23	2.42
	Inshore	Inshore	Coexist	Offshore
0.9	4.56	4.81	4.92	
	4.56	3.60	3.02	
	1.93	2.25	2.42	
	Inshore	Coexist	Offshore	
0.8	4.82	4.92		
	3.28	3.02		
	2.29	2.42		
	Coexist	Offshore		
0.7	4.92			
	3.02			
	2.42			
	Offshore			

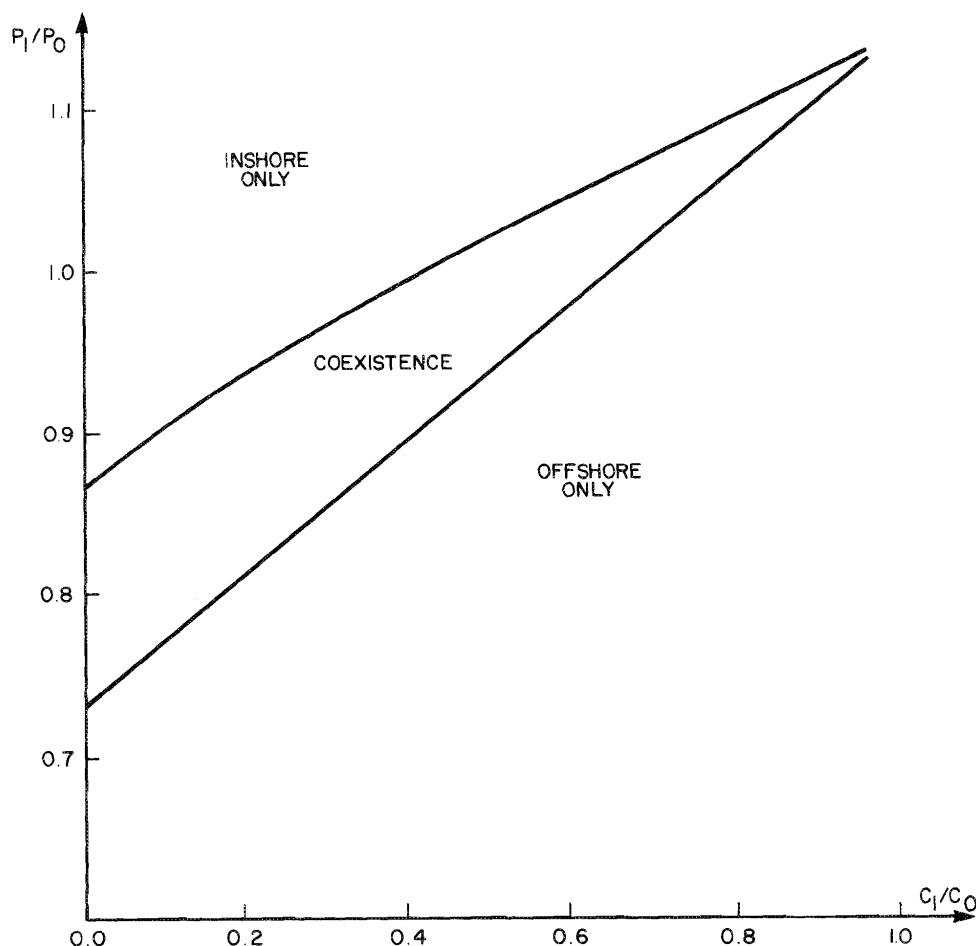


FIG. 5. Nature of the optimal equilibrium as a function of the price ratio p_1/p_0 and the cost ratio c_1/c_0 . Model parameters used to obtain these results are described in the text.

the inshore stock. With this analogy, the result of equation (26) seems reasonable: the larger is β , the greater is the effect of the offshore "fish bank," and the lower is the effective discount rate $i - \beta$. It should be noted that β also enters equation (26) through the term $X = (1 - \beta)^{-1}F(E_1)$, but the effect is still to make management more conservationist.

The "inshore only" result is shown diagrammatically in Fig. 4c, where the coexistence optimality curves ϕ_0 and ϕ_1 intersect below the constraint $E_0 = (1 - \beta)^{-1}F(E_1)$. Hence, the optimal escapements $((1 - \beta)^{-1}F(E_1^*), E_1^*)$ given by (26) lie on this constraint curve, at some point Q .

The Role of Biological and Economic Parameters

From both the dynamic analysis and the equilibrium results, clearly price and cost differentials are major factors in determining the optimal balance between inshore and offshore fisheries. Indeed the common situation in which higher prices for fish caught offshore compete with lower inshore fishing costs leads to ongoing debates between gear types. To examine these price and cost questions in more detail, we turn now to a numerical analysis of equilibrium stock and escapement levels when the economic parameters are varied. Specifically, for fixed offshore parameters p_0 and c_0 , the price ratio p_1/p_0 and the cost ratio c_1/c_0 have been varied, and the values of the optimal equilibrium biomass \bar{X} and escapements \bar{E}_0^* and \bar{E}_1^* were determined for each parameter combination. Table 1 shows a set of typical

results based on a Beverton–Holt stock–recruitment function

$$F(S) = \ell_1 S + \frac{aS}{b + S}$$

with the following arbitrarily chosen parameter values: $p = 1.0$, $c_0 = 1.0 \times 10^6$, $\alpha = 0.9$, $\ell_0 = 0.6$, $\ell_1 = 0.3$, $\sigma = 0.8$, $a = 7 \times 10^6$, and $b = 2 \times 10^6$. For each pair $(p_1/p_0, c_1/c_0)$ a triplet of numbers $(\bar{X}, \bar{E}_0^*, \bar{E}_1^*)$ is given, along with an indication of the optimal fleet composition. These results are displayed graphically in Fig. 5, which shows regions of coexistence and of exclusion. Similar results were obtained for other economic parameter values. Note that if the inshore fishery is excluded the equilibrium levels $(\bar{X}, \bar{E}_0^*, \bar{E}_1^*)$ are independent of the inshore parameters p_1 and c_1 .

As expected, the inshore fishery dominates when p_1/p_0 is relatively high and c_1/c_0 is low, while operation of the offshore fishery alone is optimal when the reverse is true.

Particularly interesting is the intermediate region of coexistence between the inshore and offshore fisheries. If inshore fishing costs are very low, coexistence is optimal for a fairly substantial range of the inshore/offshore price ratio. If the inshore unit cost c_1 is closer to the offshore cost c_0 , coexistence is constrained to an increasingly narrow range of the price ratio p_1/p_0 . Figure 5 shows this result for cost ratios in the range $0 \leq c_1/c_0 \leq 0.95$.

Note that for the set of parameter values used here, when the cost ratio c_1/c_0 is large (inshore costs comparable with offshore

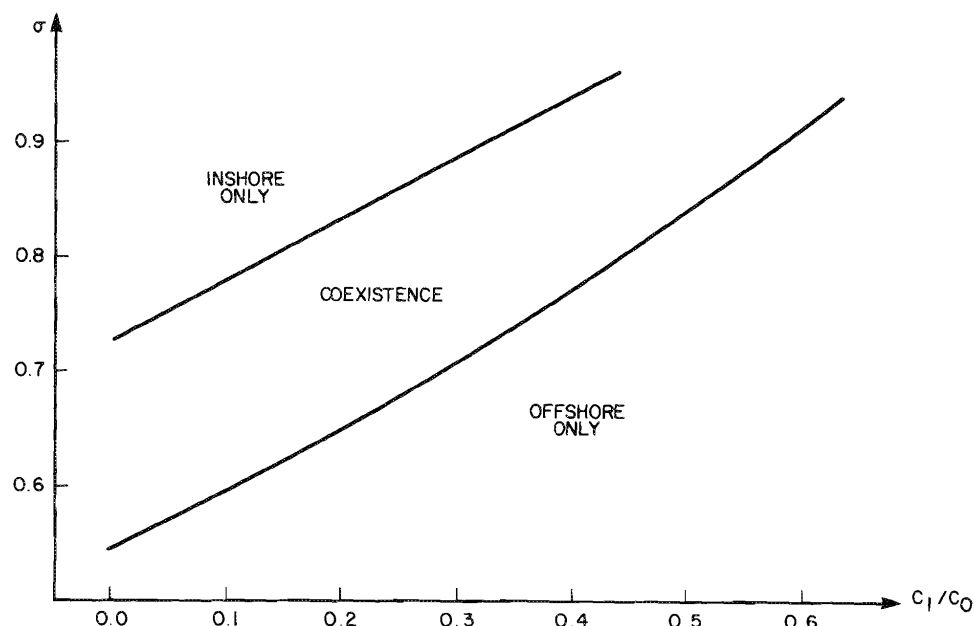


FIG. 6. Nature of the optimal equilibrium as a function of the spawning fraction σ and the inshore/offshore cost ratio c_1/c_0 (with p_1/p_0 fixed at 0.9). Regions of coexistence and exclusion depend in addition on other model parameters, which are described in the text.

costs), coexistence is optimal only when the inshore price actually exceeds the offshore price. Conversely, if $p_0 = p_1$, exclusion of the inshore fishery is optimal whenever $c_1/c_0 > 0.65$. The reason for this is that the greater availability of fish offshore can, up to a point, more than compensate for higher fishing costs. Of course, quantitative results will depend on the actual values of the biological parameters β and σ , but qualitative results seem robust to the choice of parameters.

Note also that in Table 1 the equilibrium fish stock \bar{X} and the equilibrium spawning escapement \bar{E}_1^* both decrease as the inshore fishery becomes more profitable. When high values of the price ratio p_1/p_0 combine with low values of the cost ratio c_1/c_0 , the inshore fleet predominates, and harvests at a rate high enough to reduce both \bar{X} and \bar{E}_1^* . With a low price ratio or a high cost ratio, the offshore fleet is dominant.

The biological parameter σ , representing the fraction of the total offshore fish population that moves inshore to spawn in any given year, appears to play a key role in determining the optimal equilibrium configuration in a sequential fishery. This parameter is a measure of the age structure occurring in the "vulnerable" fish population. In the case of pink salmon, all vulnerable fish in the offshore pool move inshore to spawn, so that $\sigma = 1$. Chinook salmon and herring, on the other hand, are harvested by the offshore fleet as both mature and immature fish, and hence a fraction of the offshore pool does not spawn ($\sigma < 1$).

Figure 6 shows the role of σ in determining the outcome (coexistence or exclusion) in the optimal equilibrium configuration. Although all fishery parameters interact to affect this outcome, the inshore unit cost c_1 has been used here as an indicator of the relative profitability of the inshore and offshore fisheries (holding p_1 , p_0 , and c_0 constant). Figure 6 depicts, then, the interaction between the biological parameter σ and the economic parameter c_1 , with other parameters fixed as above.

One can see that the relative advantage of the inshore fishery increases as σ rises. When σ is small, the inshore fishery should be excluded, while at high values of σ the offshore fleet should not operate (unless inshore costs are high). There is a critical

value of σ (in this case $\sigma_c = 0.54$) below which the inshore fleet is excluded even if its operating costs are negligible. Likewise there is a value of c_1 above which inshore costs are prohibitive even if all offshore fish spawn each year ($\sigma \equiv 1$), and again only the offshore fishery should take place. As in Fig. 5, a band of (c_1, σ) combinations exists for which coexistence of the two fleets is optimal.

These results reflect the basic structure of the sequential fishery. When σ is small, inshore costs per unit harvest are relatively high, simply because fewer fish move inshore (i.e. c_1/x is large if x is small). Therefore there is little incentive to have an inshore fishery. Even if σ is large, however, the marginal offshore profit from a unit harvest, $p_0 - c_0/x$, may exceed the inshore equivalent, $p_1 - c_1/x$, if the parameter c_1 is large; in such cases the inshore fleet will again be excluded. It should be noted that this interaction also depends on the relative magnitudes of the inshore and offshore survival rates, ℓ_1 and ℓ_0 , but these complications will not be addressed here.

To summarize the role of economic and biological parameters, an examination of the price and costs ratios, p_1/p_0 , and c_1/c_0 , together with the spawning fraction σ , appears sufficient to analyze the tradeoff involved in optimizing a sequential fishery.

Fishery Regulation

We have shown that optimal management of a structured sequential fishery can be achieved through controlling the escapement of fish from each fishery along the "gauntlet." Implicitly, this involves managing both the final spawning escapement and the allocation of available harvest between participating fleets. Traditionally, these two management aims have been treated separately, the former as "biological management" and the latter as a form of "political management" designed to maintain some equitable level of satisfaction (or dissatisfaction) amongst the relevant user groups. Our results show that this separation is not valid for sequential fisheries in

which competing fleets have differing prices, costs, and impacts on the fish stock. In such situations, determinations of optimal spawner levels and optimal catch allocations are tied together; arbitrary changes in catch allocation will typically produce suboptimal utilization of the resource.

A number of regulatory strategies exist for achieving optimal management of a sequential fishery. In-season catch monitoring can be used together with fishery closures, in order to achieve escapement goals in each fishery. This approach involves direct regulation and information requirements that can be expensive, but has the advantage of permitting the in-season incorporation of new information about stock size. The setting of fleet-specific catch quotas may be a promising method for simultaneously achieving overall catch level and the desired allocation of this catch. This regulatory method is particularly suited to management of a stable equilibrium fish stock; the presence of extensive year-to-year fluctuations would necessitate in-season updating of stock size estimates and hence of appropriate catch quotas.

A further option for fishery management may be the setting of separate landings taxes on each of the sequential fisheries. We consider this indirect form of economic regulation in somewhat more detail, since it fits well within the framework of this paper, although as with other regulatory measures, taxes must reflect annual fluctuations together with other uncertainties, and hence are by no means a simple solution.

An optimal taxation scheme changes the effective cost/price ratios to enable a socially optimal exploitation pattern to be realized through open access. Suppose taxes are imposed at the rates of t_0 dollars per fish landed offshore and t_1 dollars per fish landed inshore. Then the effective prices to the fishermen become $p_0 - t_0$ and $p_1 - t_1$, respectively. We wish to choose values of t_0 and t_1 to make the bionomic equilibrium derived earlier coincide with the socially optimal equilibrium. By examining separately the three cases involving coexistence or exclusion, corresponding to Fig. 4a, 4b, and 4c, specific taxation results can be obtained for each possible pattern of exploitation. Graphically the objective is to make the equilibrium point Q in Fig. 3 coincide with its optimal counterpart in Fig. 4.

An examination of these figures shows that, given the actual optimum escapements \hat{E}_0^* and \hat{E}_1^* together with the corresponding fleet configuration, sufficient taxes to optimize the equilibrium open-access fishery are $t_0 = p_0 - c_0/\hat{E}_0^*$ and $t_1 = p_1 - c_1/\hat{E}_1^*$. Although these are not necessarily the optimal *dynamic* taxes (see Clark 1976, p.116), they do hold in equilibrium and they are sufficient to drive an unregulated open-access fishery to the optimum.

Discussion

We have provided an analytical framework for examining optimal fleet and catch allocation between the inshore and offshore components in sequential "gauntlet" fisheries. The model we analyzed is based on the assumption that the offshore fishery harvests from the whole fish population, while the inshore fishery exploits only fish that are heading towards their spawning grounds. This is the case, for example, in many salmon and herring fisheries. Although we have referred to "inshore" and "offshore" fisheries, it was pointed out that the *timing* of the sequential fisheries is of importance, not the geographical location; indeed, the fisheries may occur in precisely the same location. Age structure is implicitly included in the model, and indeed is a fundamental feature, since adult spawners and nonspawning juveniles play very different roles. By a judicious choice of the biological parameters, the model

can also deal with two fisheries that act sequentially on the same mature fish stock, but that use different types of gear. However, this involves the loss of the age-structure component of the model, and hence is useful only for such cases as the British Columbia pink salmon fishery, where juvenile fish are not generally relevant to the fishery.

Economic factors have been summarized in the form of price and fishing cost parameters. While ignoring possible price elasticity and nonlinear costs, the model is sufficient to show the fundamental economic tradeoffs which determine optimal fleet allocation. In particular, the two cost/price ratios c_0/p_0 and c_1/p_1 determined the open-access bionomic equilibrium. Numerical results involving only variation of the price ratio p_1/p_0 and the cost ratio c_1/c_0 were adequate to illustrate the three possible optimal equilibrium fleet allocation scenarios: exclusion of the inshore fleet, coexistence of the inshore and offshore fisheries, and exclusion of the offshore fleet. On the other hand, analytic results showed that the full socially optimal solution depends in a more complicated manner on the prices and costs.

The social optimum is also strongly influenced by biological parameters, in particular the fraction of the offshore stock that spawns each season (σ). When σ is small, there should be no inshore fishery, while if σ is large an inshore fishery may be desired, and indeed the offshore fishery may be excluded in equilibrium.

The possible optimality of coexistence between inshore and offshore fishing fleets has been emphasized in this paper. Numerically, coexistence will occur only for a fairly narrow range of price and cost parameters. Furthermore, from our results it appears that coexistence will arise in the optimal solution only if the spawning fraction σ is sufficiently large; analytic results proved this to be the case also for the open-access equilibrium.

Analysis of coexistence versus exclusion under optimal exploitation can depend substantially on whether the dynamic or the equilibrium solution is being considered. Even if the optimal equilibrium configuration involves only the inshore fishery, a full dynamic solution may show that the offshore fleet is still desired in years of high fish stock abundance. For example, variable offshore costs per unit harvest may be sufficiently low to warrant the operation of an offshore fishery, which would then be phased out as stocks decline to equilibrium. In such cases, the offshore fishery would absorb catch variability, while the inshore catch remains relatively stable. In fact, in some situations an exceptionally good year may make it desirable to bring a previously excluded offshore fleet back into the fishery. However, such dynamic aspects of inshore/offshore allocation are complicated if fleet capital is nonmalleable (Charles 1983a; Clark et al. 1979), so that fishing boats have no alternative uses outside the fishery. In such cases, if these vessels are restricted from operating in the fishery, opportunity costs of this idle capital and physical depreciation of the vessels may make the decision to exclude one gear type from the fishery essentially irreversible. Furthermore, in the absence of an offshore fleet, the possible optimality of introducing such a fleet for a temporary period of "mining" an abundant resource will depend on a full analysis of joint fleet investment and harvest management. With stochastic resource fluctuations, this becomes a complex problem of investment under uncertainty (Charles 1983b). The extent of capital malleability will determine to a large degree whether the fleet mix will respond to such stochastic effects over time or whether the equilibrium configuration, once attained, will predominate in the fishery.

Finally, we discussed various regulatory methods for managing sequential fisheries. We found that taxes can be used to adjust the open-access equilibrium fishery to match the social optimum. Two distinct taxes are required on the inshore and offshore catches separately; in equilibrium, these optimal taxes are determined in terms of E_0^* and E_1^* , the optimal equilibrium escapement levels. Such taxes (or other regulatory measures) could be applied, for example, in the British Columbia herring fishery, where the "offshore" food fishery is not desired from the point of view of economic efficiency, but in fact exists primarily because it is the first fishery available each season. To maximize total rents, this fishery perhaps should be foregone entirely, but the optimal decision for many individual fishermen is to take part in both the offshore and inshore fisheries. Appropriate fishery regulation may serve as a tool to transform this coexisting fishery into an inshore-only fishery.

Of course, considerations other than simple rent maximization can play an important role in determining allocations between fisheries. In some cases, the offshore and inshore fisheries may appeal to completely separate markets, so that by maintaining both fisheries, at least to a limited extent, fisheries management can achieve a measure of demand (and hence fisherman income) stability at the expense of foregone rents. The value judgements involved in making such multiobjective decisions can best be left to managers and politicians, but the analysis presented here provides a methodology for measuring losses in economic rents due to alternative (economically suboptimal) catch allocations between fleets in a sequential fishery.

Possible applications of the model to salmon and herring fisheries on the Pacific coast of Canada have been discussed previously. Problems of gear conflict are also prevalent on Canada's Atlantic coast. The general form of analysis presented here could be applied, for example, to the Newfoundland fishery on northern cod (*Gadus morhua*), where small inshore boats and large "factory-type" offshore vessels currently compete and coexist (Munro and McCorquodale 1981). This fishery has been well studied, but not yet within the context of a full bioeconomic optimization analysis. Inshore/offshore fisheries are common as well in developing countries, where typically the needs of small-scale domestic fishermen must be balanced with

the benefits of agreements allowing distant water fleets to harvest local fish stocks (A. T. Charles, unpubl. data).

A full analysis of uncertainties, nonlinearities, and mixed stock problems may be required in order to determine the optimal fleet mix for specific real-world fisheries. Nevertheless the general results obtained here regarding coexistence, competition, and optimal catch allocation between inshore and offshore fisheries are likely to be quite robust in practice.

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